Calorimetry
in particle physics experiments

4. Calorimeter Performance: Energy Resolution

Course roadmap

- **Week 1 (Foundations)**
  - ✓ Lecture 1: Why calorimetry?
  - ✓ Lecture 2: EM shower physics
- **Week 2 (Physics depth)**
  - ✓ Lecture 3: Hadronic shower physics
  - ✓ Lecture 4: Energy resolution from first principles
- **Week 3 (Technology)**
  - ✓ Lecture 5: Calorimeter Technologies (real-life EM and Hadronic calorimeters)
  - ✓ Lecture 6: Calorimeter Design
- **Week 4 (Systems & Future)**
  - ✓ Lecture 7: Signal chain, readout, calibration
  - ✓ Lecture 8: Future calorimetry

Today's Lecture

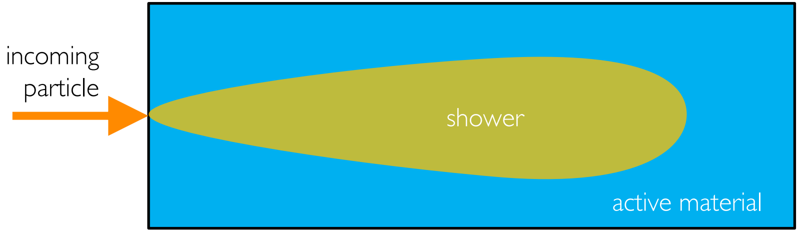
- **Week 2 (Physics depth)**
  - ✓ Lecture 3: Hadronic shower physics
  - ✓ Lecture 4: Energy resolution from first principles
    - *4.1 From Shower Physics to Calorimeter Design*
    - *4.2 The Three-Term Energy Resolution Formula*
    - *4.3 The Stochastic Term: Sampling Fluctuations*
    - *4.4 Noise Term, Constant Term & Leakage*
    - *4.5 Hadronic Energy: e/h & Compensation*
    - *4.6 Position Measurement & Time Resolution*

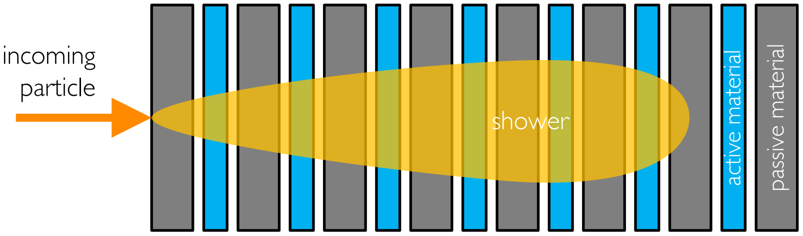
4.1

From Shower Physics to Calorimeter Design

Instrumenting a shower: two strategies

- We now understand shower development… how do we *instrument* it to measure the energy?

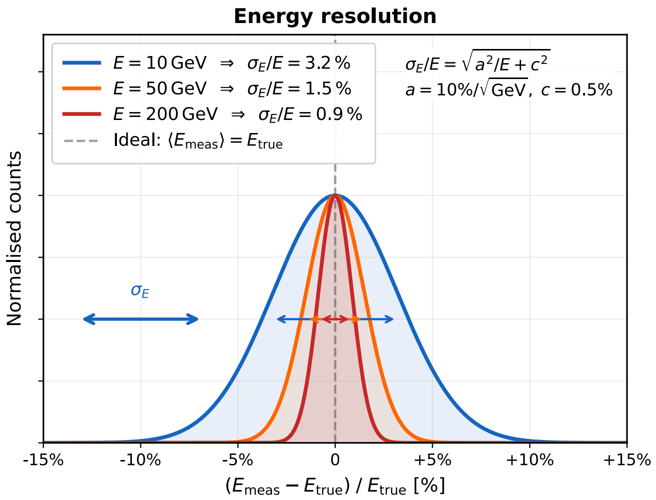


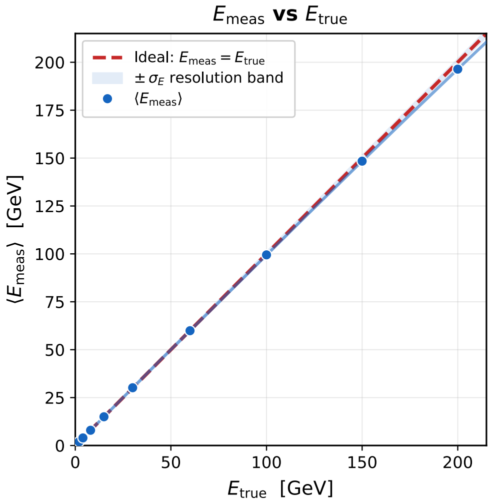


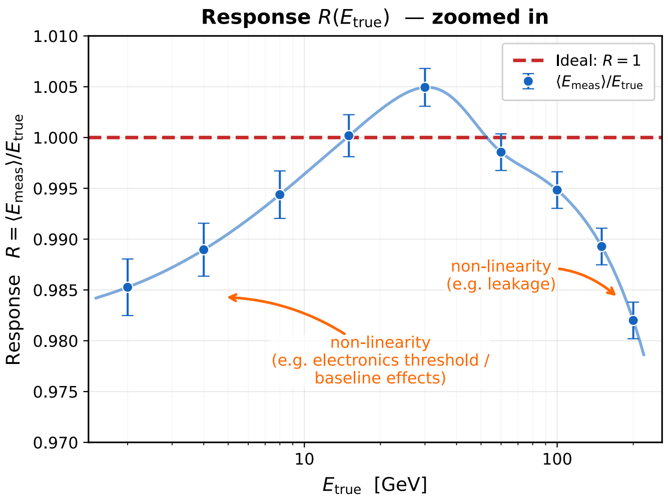
- **Homogeneous calorimeter: entire volume is simultaneously absorber AND active medium**
  - ✓ Signal produced in the shower; no energy is "lost" in passive material
  - ✓ Best achievable stochastic term (only intrinsic fluctuations): best energy resolution
  - ✓ Very expensive; exclusively used for EM calorimeters
  - ✓ Examples
    - CMS ECAL (PbWO4 crystals)
    - NA48/NA62 (liquid krypton)
    - Belle II ECL (CsI)

- **Sampling calorimeter: passive absorber plates interleaved with thin active readout layers**
  - ✓ Only fraction f_{samp} of shower energy is sampled, absorber energy is lost
    - Typically a few percent (for gas detectors even only ~10^{-5})
  - ✓ Finite sampling introduces fluctuations (dominant contribution to stochastic term)
  - ✓ Cheaper, more flexible, scalable to large volumes (e.g. for HAD calorimeter)
  - ✓ Examples
    - ATLAS LAr accordion (Pb + LAr)
    - CMS HCAL (brass + plastic scintillator)

Instrumenting a shower: response resolution and linearity







- **Resolution**
  - ✓ Response is affected by event-to-event variations of the signal
  - ✓ Different (concurrent) sources of fluctuation affect response differently at different energies
- **Linearity**
  - ✓ Average calorimeter signal per unit of deposited energy
  - ✓ Ideally *proportional* to input energy: a linear calorimeter has a constant response (after *calibration*)
  - ✓ In general: EM calorimeters are (mostly) linear, hadronic calorimeters are not
  - ✓ Compensation might help to restore response linearity

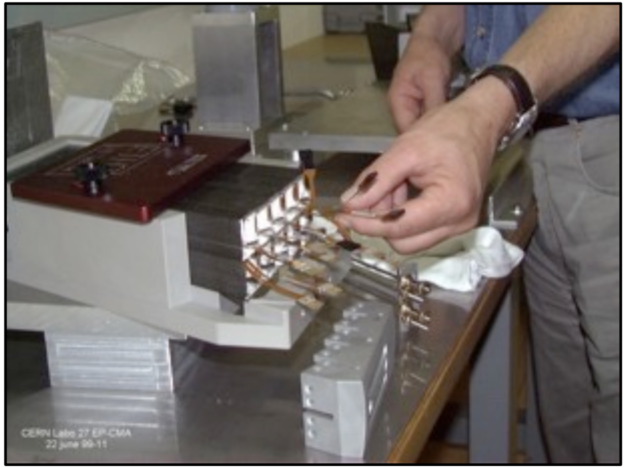
Active media: what signal do we actually read out?

- Three fundamental signal types, to be turned into a *recordable* signal (electronics):

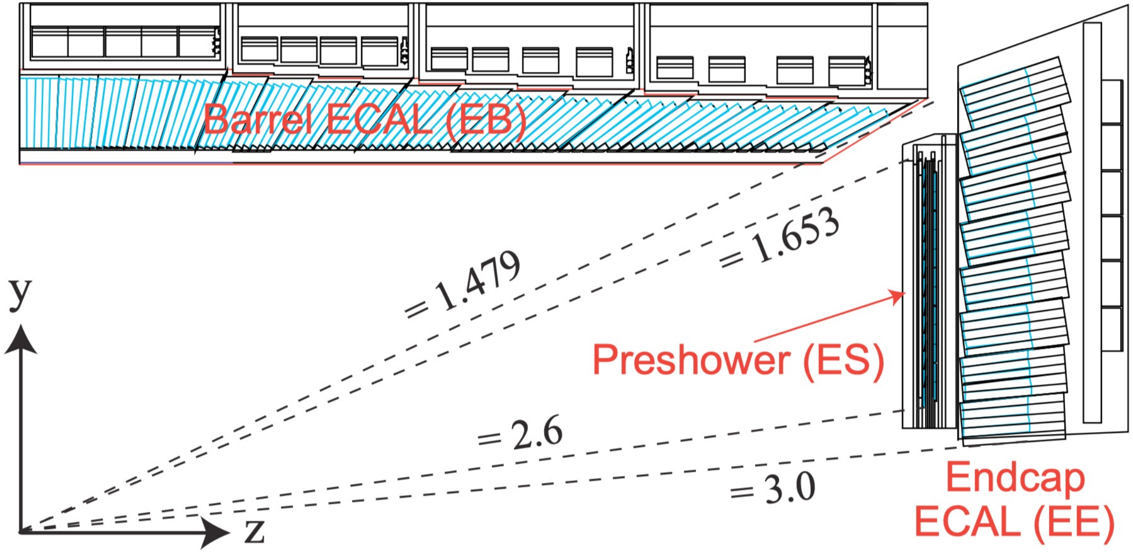
Signal	Detector	Mechanism	Characteristics
Ionization charge	Noble liquids (LAr, LKr, LXe) Silicon sensors	Charged particles ionize the medium; drift field collects e/ions onto electrodes	• Excellent uniformity • Intrinsically radiation-hard • Requires cryostat (noble liquids), or thin-gap geometry (Si)
Scintillation light	Inorganic crystals ($PbWO_4$, BGO, NaI, CsI) Plastic scintillator, WLS fibres	Shower excites crystal lattice or organic molecules; photons collected by photodetector (APD, SiPM, PMT)	• Key figures: light yield [ph/MeV], decay time [ns], radiation hardness • Vary enormously across materials
Cherenkov light	Lead glass Quartz fibres Small component in $PbWO_4$	Charged particles with $\beta > 1/n$ emit coherent Cherenkov radiation	• Very fast (instantaneous) • Only from relativistic particles ($\beta > 1/n$): suppresses low-energy hadronic component

Full technology comparison (light yield, decay time, radiation hardness, cost, …) → Lectures 5 and 6

Example of a homogeneous calorimeter: CMS ECAL

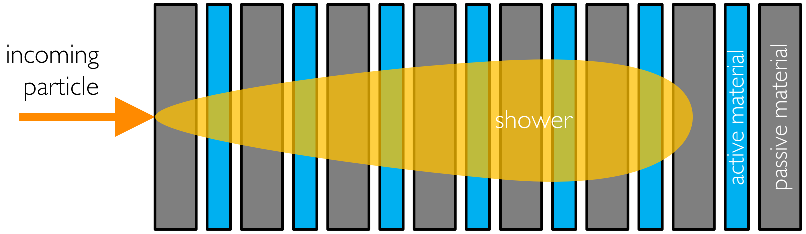


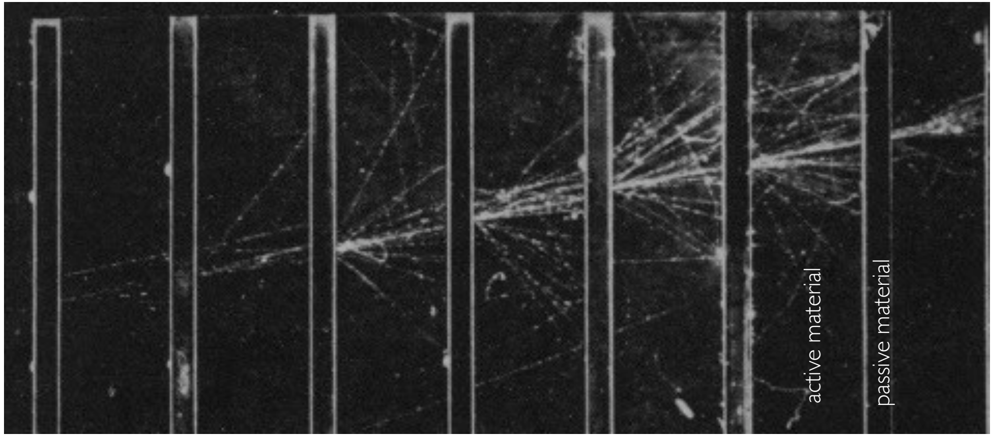
- Scintillator: $PbWO_4$ [Lead Tungstate]
- Photosensor : APDs [Avalanche Photodiodes]
  - ✓ ~70000 crystals
  - ✓ ~4500 photoelectrons/GeV
    - $PbWO_4$ ~100 photons/MeV; after APD yield ~4.5 pe/MeV



Passive (and active) media for sampling calorimeter

- Principle of sampling calorimeters: alternating layers of absorber and active material
- Passive absorbers principle responsible for shower development and cascade of secondaries
- Absorber (passive) materials
  - ✓ High density, high Z
    - Iron (Fe)
    - Lead (Pb)
    - Uranium (U)
      - also for compensation in HCAL
- Active materials
  - ✓ No need to be as high density as for most homogeneous calorimeters (cheaper)
    - Plastic scintillator
    - Liquid ionization chamber
    - Gas detectors

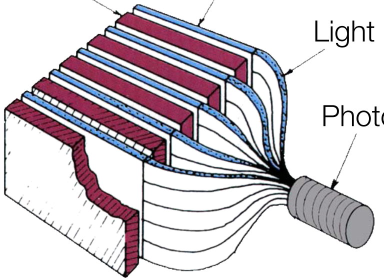




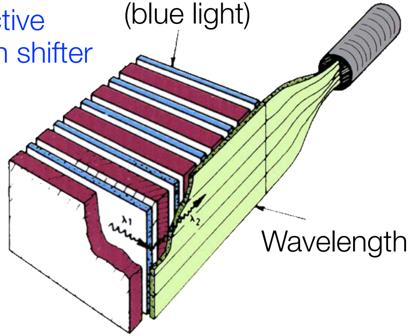
Electromagnetic shower

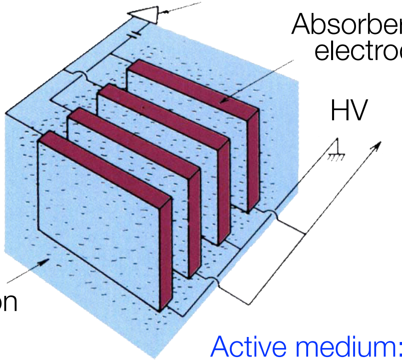
Possible sampling calorimeter setups

Scintillators as active layer;
signal readout via photo multipliers



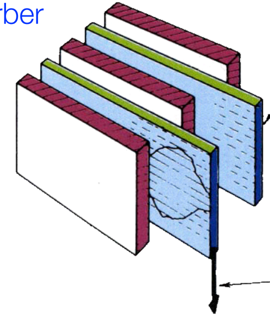
Scintillators as active layer; wave length shifter to convert light



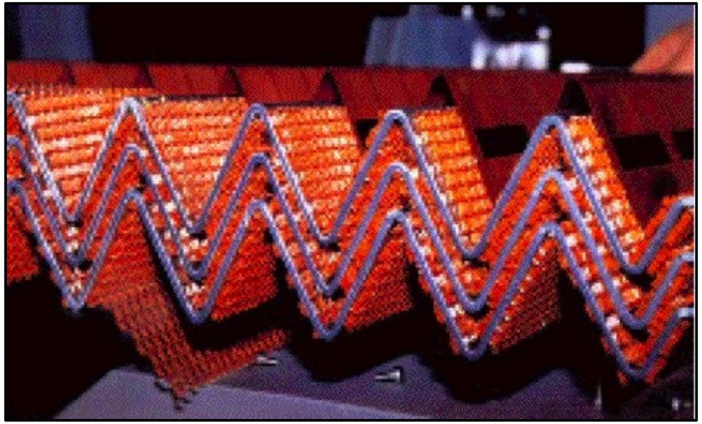


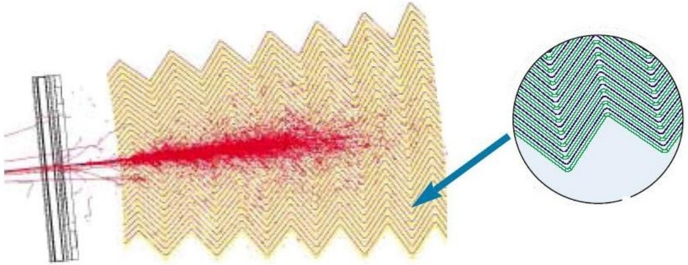
Active medium: LAr; absorber embedded in liquid serve as electrods

Ionization chambers between absorber plates

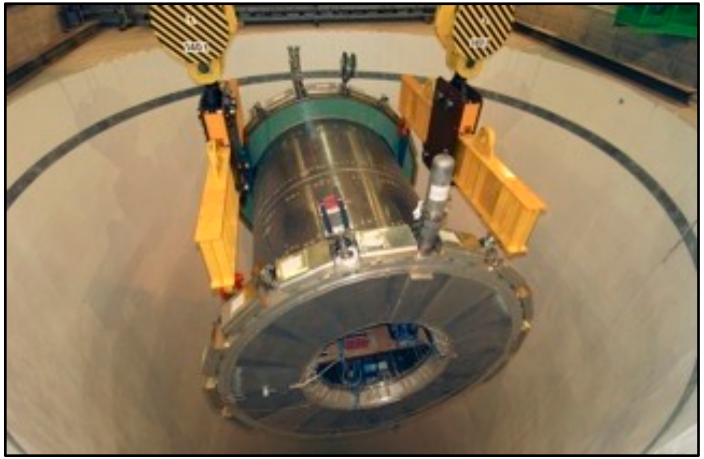


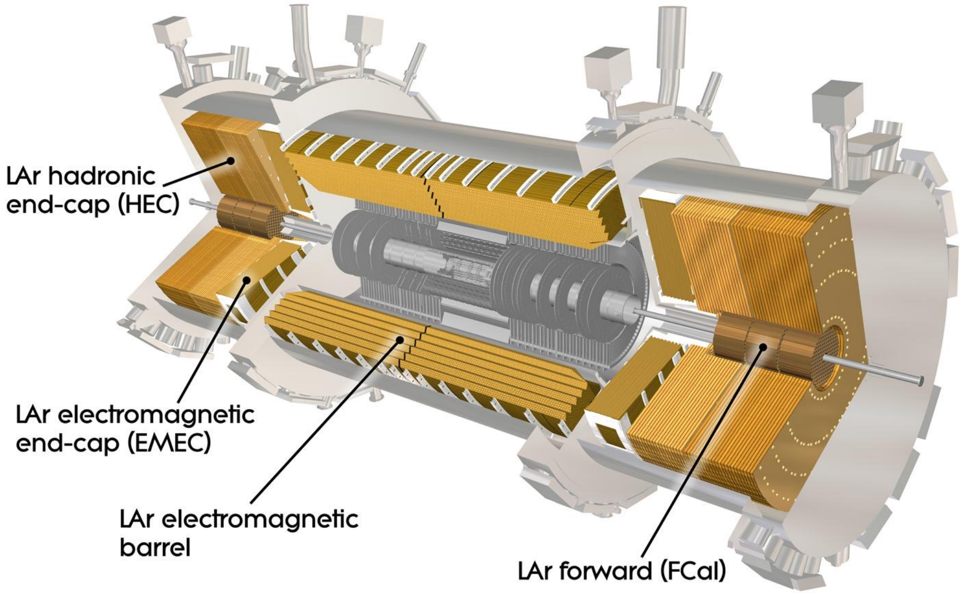
Example of a sampling calorimeter: ATLAS LAr





- Pb absorbers
- 2 mm LAr gap + 2 kV HV (ionization current)
- ~200000 channels

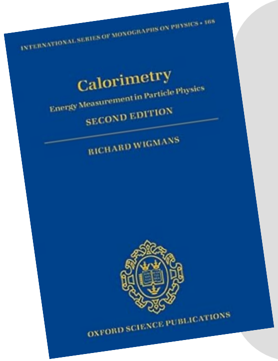




What a perfect HCAL would need to do?

- Answering the bridge question from Lecture 3:
  - ✓ A. Sufficient depth: ~ 10 λ_{int}
    - will derive the containment requirement shortly
  - ✓ B. Equal response to EM and hadronic → e/h = 1 (compensation)
  - ✓ C. Good sampling: thin active layers, many samples → minimize stochastic term
  - ✓ D. Fast readout: shaping time captures most of signal within LHC 25 ns window

- These four requirements translate directly into the three-term resolution formula…



> "If one uses a sampling calorimeter in which the sampling fraction is not optimised for compensation, then one wastes money…
>
> \- R. Wigmans

4.2

The Three-Term Energy Resolution Formula

The Three-Term Energy Resolution Formula

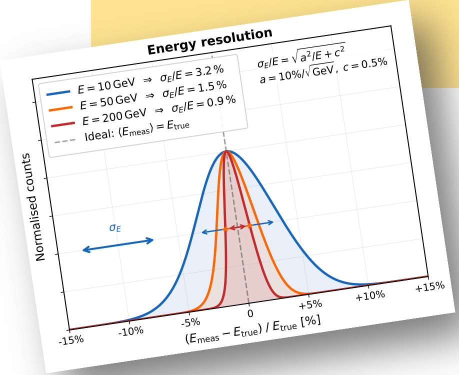
relative energy resolution

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus \frac{b}{E} \oplus c$$

"stochastic" term

"noise" term

"constant" term



$$a \oplus b = \sqrt{a^2 + b^2}$$

terms uncorrelated, propagate and sum as Gaussian errors

Origin of resolution formula: track-length integral

- Rossi track-length argument: total ionization by particle in a shower is proportional to particle initial energy E_0
  - ✓ For an EM shower: total charged-particle track length

$$T_0 \sim X_0 \frac{E_0}{E_c} \qquad [\mathrm{g\ cm^{-2}}]$$

  - ✓ Each charged particle deposits ionization dE/dx ~ const (MIP-like below E_c)
  - ✓ Total signal = sum of contributions from all track segments

- For a sampling calorimeter: only a fraction f_{samp} of track length is sampled
  - ✓ Signal proportional to E_0, but with fluctuations from finite number of samples

- Several independent sources of fluctuation…

Energy resolution fluctuation sources

- Calorimeter energy resolution determined by fluctuations
  - ✓ Homogeneous calorimeters:
    - Shower fluctuations
    - Photo-electron statistics
    - Shower leakage
    - Instrumental effects
      - noise, light attenuation, non-uniformity, …

  (Shower fluctuations, Photo-electron statistics, Shower leakage} Quantum fluctuation)

Quantum fluctuations	$\sim {}^{1}\!/\!\sqrt{E}$
Shower leakage*	$\sim \mathrm{const}$
Electronic noise	$\sim {}^{1}\!/\!E$
Non-uniformity	$\sim \mathrm{const}$

- In addition, for sampling calorimeters:
  - ✓ Sampling fluctuations
  - ✓ Landau fluctuations
  - ✓ Track length fluctuations

Sampling fluctuations	$\sim {}^{1}\!/\!\sqrt{E}$
Landau fluctuations	$\sim {}^{1}\!/\!\sqrt{E}$
Track length fluctuations	$\sim {}^{1}\!/\!\sqrt{E}$

$*$ longitudinal leakage fluctuates event-to-event → constant-like contribution

The three terms: physical origins

$$\frac{a}{\sqrt{E}}$$

- Quantum and sampling fluctuations
  - ✓ For homogeneous: Fano factor fluctuations in number of primary ionizations
  - ✓ For sampling: finite number of track segments sampled (dominant)
  - ✓ Also: Landau fluctuations in thin absorbers; shower-to-shower fluctuations

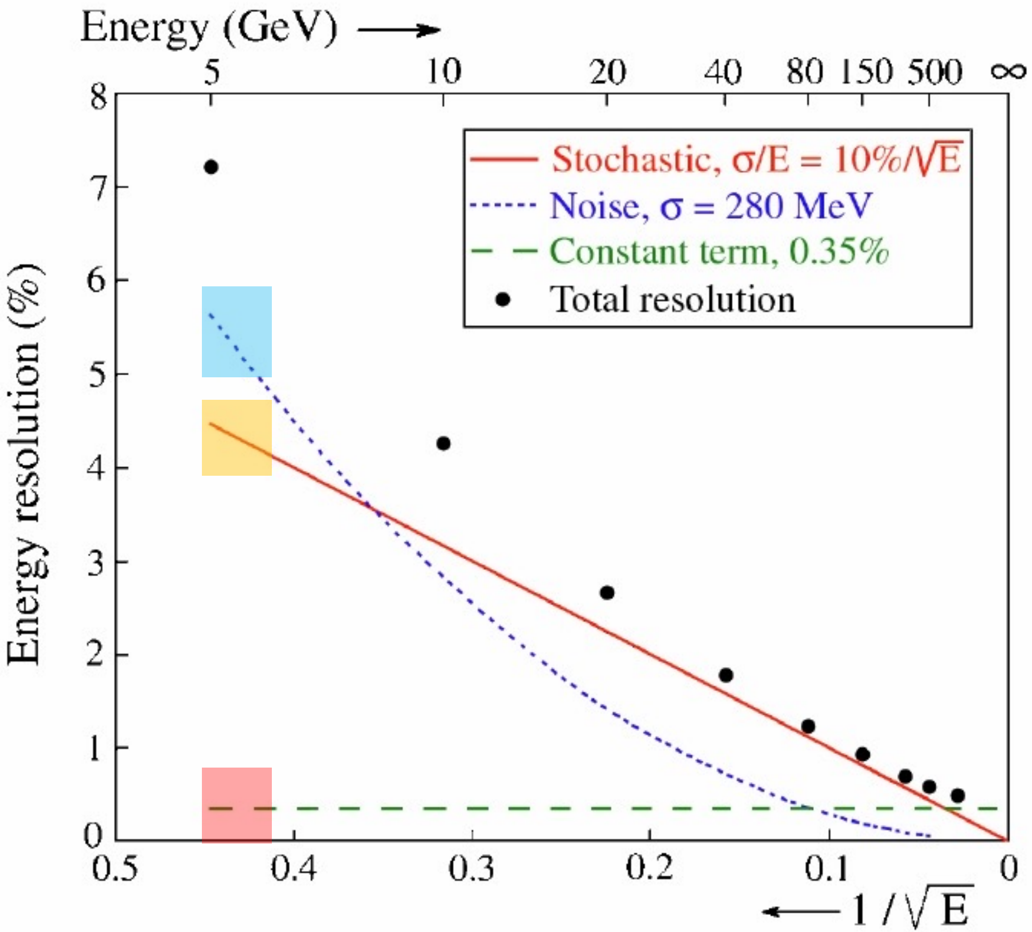
$$\frac{b}{E}$$

- Noise
  - ✓ In general does not depend on energy (i.e. constant), thus scale linearly with energy in relative energy resolution
  - ✓ Dominates at low E if b/E large
  - ✓ Typical source: readout electronics
    - Might have a "range" energy dependence
  - ✓ At LHC b also includes pile-up noise
    - Depends on clustering algorithm and PU corrections

$$c$$

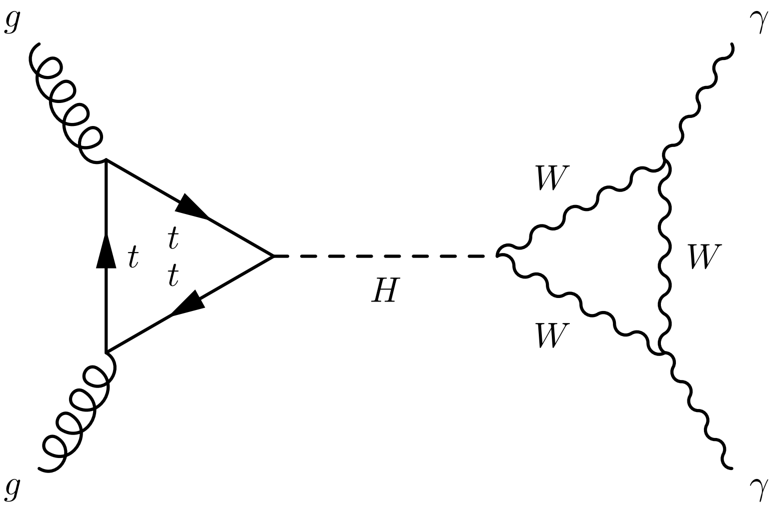
- Residual non-uniformity at all energies
  - ✓ Inter-channel response variations after calibration (residual local response variation (e.g. crystal opacity)
  - ✓ Temperature gradients, dead material, longitudinal leakage fluctuations
  - ✓ Dominates at high E
    - c = 0.5% for CMS ECAL
    - c = 0.7% for ATLAS LAr

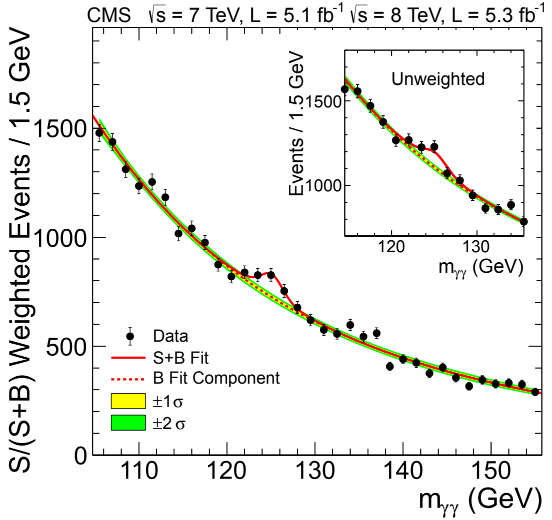
The three terms: an example



ATLAS LAr EM barrel calorimeter @ **η= 0.28**

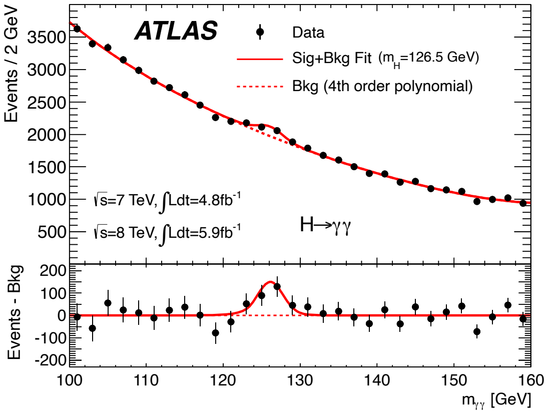
Why energy resolution matters: H → γγ as a benchmark





$$m_{\gamma\gamma} = \sqrt{2E_1^{\gamma}E_2^{\gamma}(1-\cos\alpha_{12})}$$

$$\frac{\sigma_{m_{\gamma\gamma}}}{m_{\gamma\gamma}} = \frac{1}{2}\sqrt{\left(\frac{\sigma_{E_1}}{E_1}\right)^2 + \left(\frac{\sigma_{E_2}}{E_2}\right)^2 + \left(\frac{\sigma_{\theta_{12}}}{\tan\theta_{12}}\right)^2}$$



4.3

The Stochastic Term: Sampling Fluctuations

Stochastic term for homogeneous calorimeters & Fano factor

- In ideal (homogeneous) calorimeter without leakage energy resolution only limited by statistical fluctuations of the number of shower particles...
  - ✓ Signal = total number of e-h pairs (or photons) produced in EM shower

Poisson

$$\frac{\sigma_E}{E} \propto \frac{\sigma_N}{N} \sim \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

Mean number of particles

$$N = \frac{E_0}{W}$$

W = mean energy required to produce "signal quantum"

W ~ 3.7 eV for Si
W ~ 30 eV for NaI
W ~ 100 eV for plastic scintillator

$$\frac{\sigma_E}{E} \propto \sqrt{\frac{W}{E}}$$

- Resolution actually *improves* due to correlations between fluctuations (assuming shower is completely contained): Fano factor F
  - ✓ F < 1 for semiconductors
  - ✓ F ~ 1 for scintillators
  - ✓ Example: LAr (W ~ 30 eV/pair, F ~ 0.1) → a ~ 0.3%/sqrt(E) → tiny!
    - Homogeneous calorimeters can have excellent intrinsic stochastic term

$$\sigma_N^2 = FN$$

$$\frac{\sigma_E}{E} \propto \sqrt{\frac{FW}{E}}$$

Photo-electron statistics impact on stochastic term

- For detectors for which the deposited energy is measured via light detection inefficiencies converting photons into a detectable electrical signal (e.g. photo electrons) contribute to the measurement uncertainty

$$\frac{\sigma_E}{E} \propto \frac{\sigma_{N_{\mathrm{pe}}}}{N_{\mathrm{pe}}} = \frac{1}{\sqrt{N_{\mathrm{pe}}}}$$

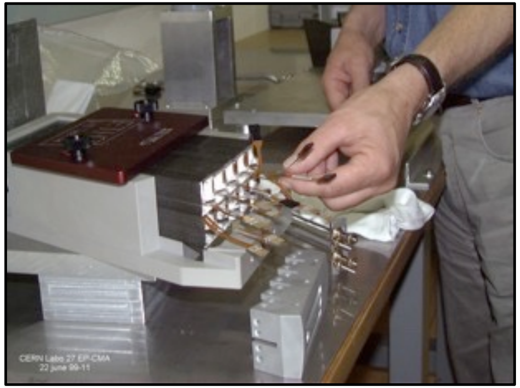
N_{pe} : number of photo electrons

- Contribution present for calorimeters based on detecting scintillation or Cherenkov light
- Important in this context
  - ✓ Quantum efficiency (QE) and gain of used photo detectors (e.g. Photomultiplier, Avalanche Photodiodes, ...)
  - ✓ Also losses in light guides and wavelength shifters ....

Exercise: compute *a* for CMS ECAL

- CMS ECAL = PbWO4 crystals read via APD
  - ✓ PbWO4 is a relatively weak scintillator. In CMS ~ 4500 photoelectrons/GeV (APD QE ~80%)
  - ✓ Fano factor F ~ 2 for crystal/APD combination in crystals
    - F ~ 1 + fluctuations in avalanche multiplication process of APD (*excess noise factor*)
- Expected stochastic term:

$$a_{\mathrm{pe}} = \sqrt{\frac{F}{N_{\mathrm{pe}}}} = \sqrt{\frac{2}{4500}} = 2.1\%$$

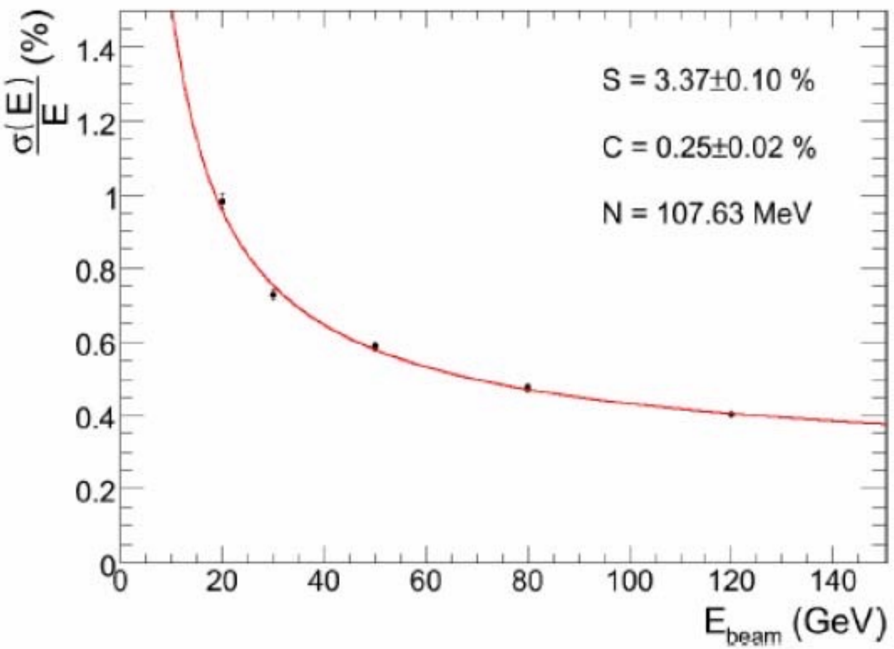


- This does not include all possible effects, e.g. lateral leakage from limited clusters of crystals (to minimize electronic noise and pile-up)
  - a_{leak} = 1.5% (Σ(5x5))
  - a_{leak} = 2% (Σ(3x3))
  - For Σ(3x3) case $a = a_{\mathrm{pe}} \oplus a_{\mathrm{leak}}$ = 2.9%

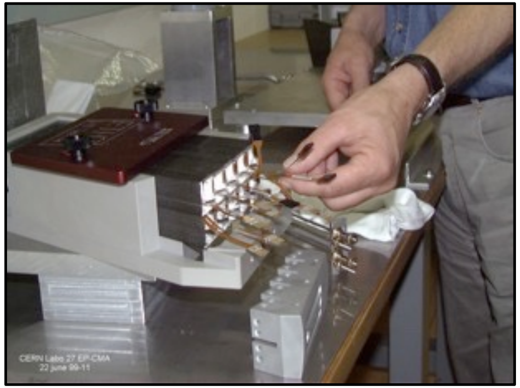
Exercise: compute *a* for CMS ECAL

$$\left(\frac{\sigma}{E}\right)^2 = \left(\frac{3.37\%}{\sqrt{E}}\right)^2 + \left(\frac{0.107}{E}\right)^2 + (0.25\%)^2$$

stoch. noise const.



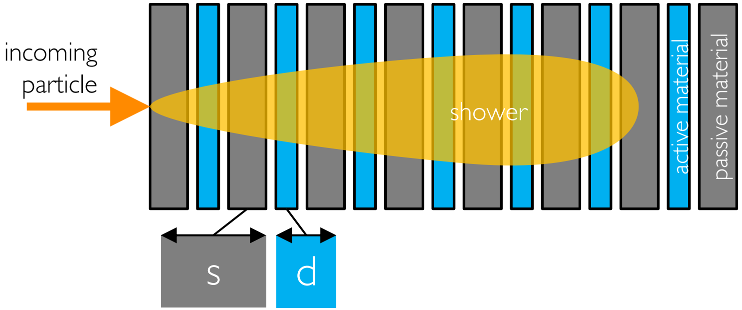
- For Σ(3x3) case $a = a_{pe} \oplus a_{leak} = 2.9\%$
- measured value: $a_{meas} = 3.4\%$



Stochastic term for sampling calorimeters: sampling fraction

- In sampling calorimeters stochastic term is dominated by *sampling* fluctuations
  - ✓ = fluctuations in the total number of electron and positron tracks crossing the active planes
- **Sampling fraction**

$$f_{\mathrm{samp}} = \frac{d\frac{dE}{dx}^{\mathrm{active}}_{\mathrm{MIP}}}{d\frac{dE}{dx}^{\mathrm{active}}_{\mathrm{MIP}} + s\frac{dE}{dx}^{\mathrm{absorber}}_{\mathrm{MIP}}}$$



$$N_{\mathrm{charged}} = \frac{f_{\mathrm{samp}} \cdot E}{d \cdot \left(\frac{dE}{dx}\right)^{\mathrm{MIP}}_{\mathrm{active}}} \propto \frac{f_{\mathrm{samp}} \cdot E}{d}$$

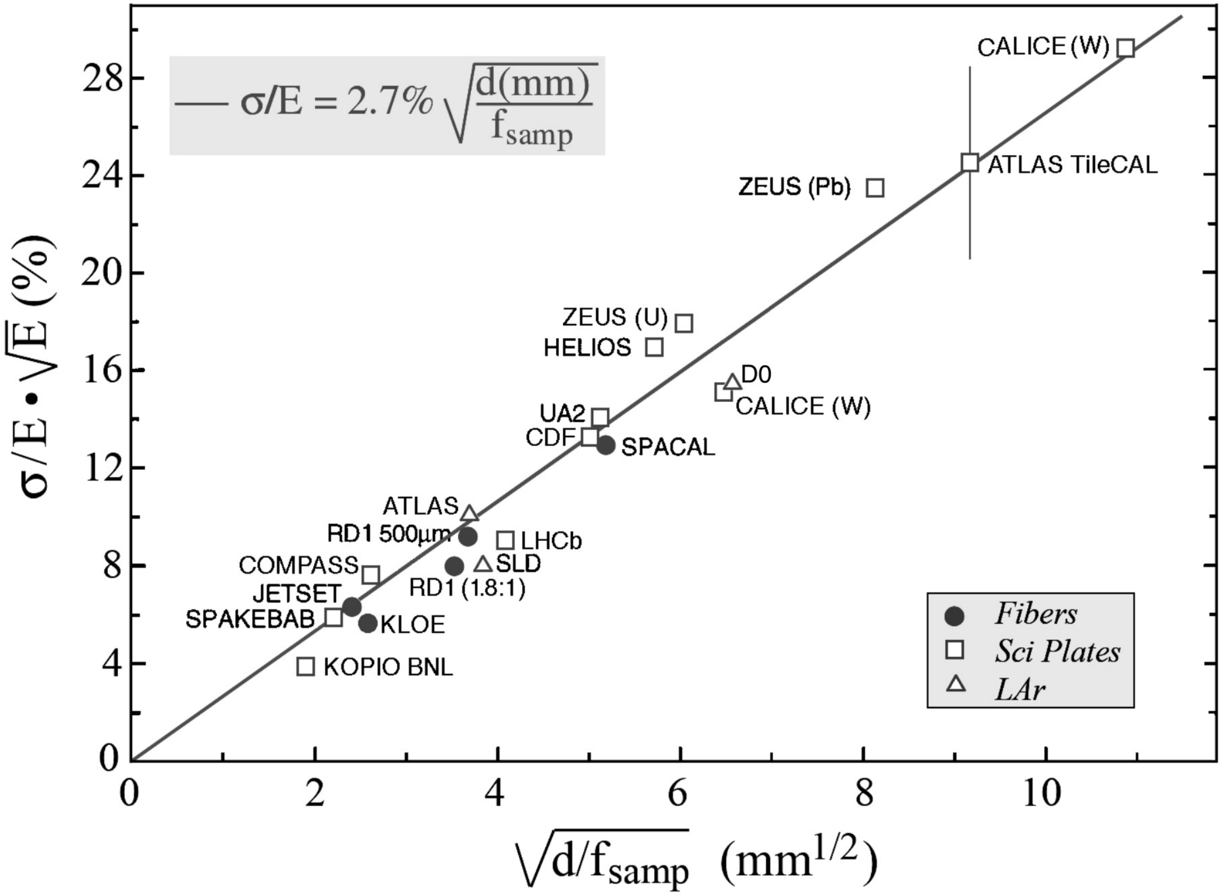
$$\frac{\sigma_E^{\mathrm{samp}}}{E} \propto \frac{1}{\sqrt{N_{\mathrm{charged}}}} \propto \sqrt{\frac{d}{f_{\mathrm{samp}}}}\frac{1}{\sqrt{E}}$$

$N_{charged}$ *= total energy deposited in active layers / energy deposited per crossing*
Number of charged particles crossing active layers increases linearly with incident energy and fineness of sampling

Resolution improves as d decreases but resolution degrades as f_{samp} decreases!

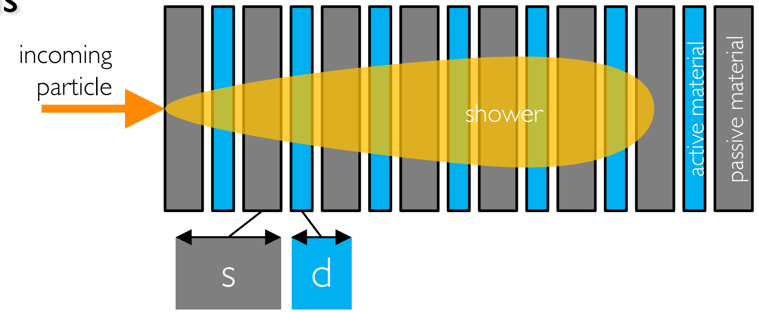
Stochastic term for sampling calorimeters vs active layer

$$\frac{\sigma_E^{\text{samp}}}{E} \propto \sqrt{\frac{d}{f_{\text{samp}}}}\frac{1}{\sqrt{E}}$$



Stochastic term for sampling calorimeters vs *passive* layer

- Energy deposition dominantly due to low energy electrons
  - ✓ Range of these electrons smaller than absorber thickness
  - ✓ Only few electrons reach active layer
- Only fraction f ~ 1/s reaches active medium



$$N_{\text{charged}} \propto \frac{E}{E_c s}$$

$N_{charged}$ = charged particles reaching active layer
E/E_c = total number of particles
s = absorber thickness in X_0

$$\frac{\sigma_E}{E} \propto \frac{\sigma_{N_{\text{charged}}}}{N_{\text{charged}}} \propto \sqrt{\frac{E_c s}{E}}$$

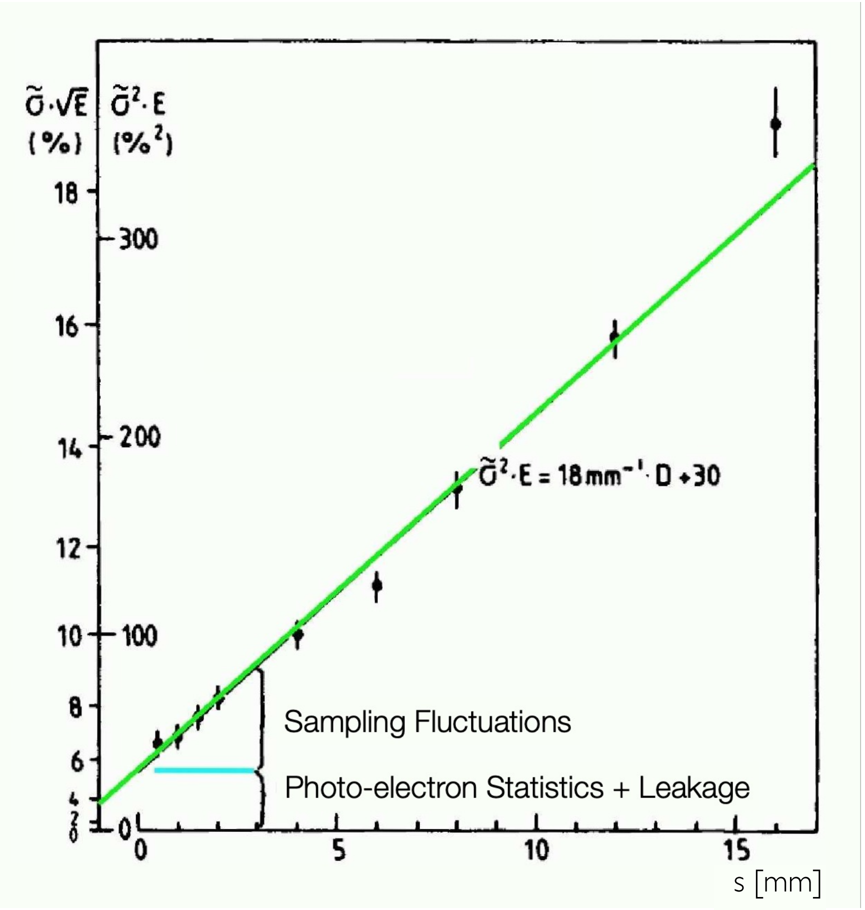
Optimization: choose small E_c (large Z absorbers) and small s (fine sampling)

$$E_c^{\text{Gas}} = \frac{710\text{ MeV}}{Z+0.92} \qquad E_c^{\text{Sol/Liq}} = \frac{610\text{ MeV}}{Z+1.24}$$

Reminder: higher-Z materials have stronger electric fields that enhance bremsstrahlung (lowering E_c)

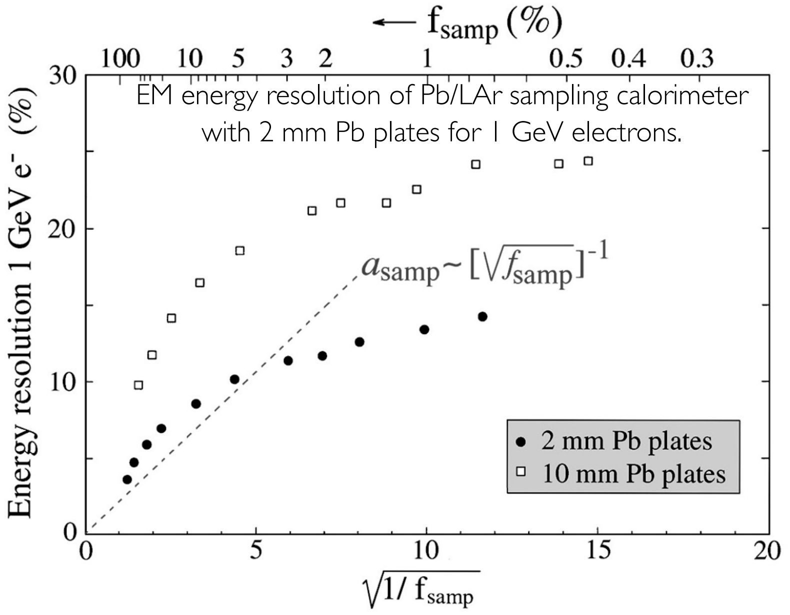
Stochastic term for sampling calorimeters vs *passive* layer

$$\frac{\sigma_E}{E} \propto \sqrt{\frac{E_c s}{E}}$$



Sampling fluctuations in extreme conditions

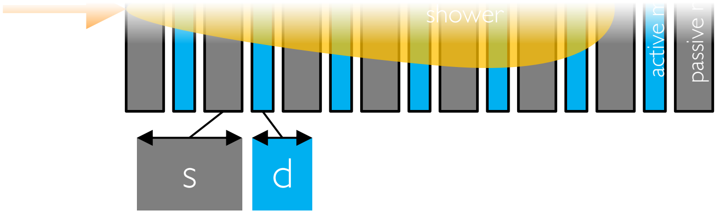
- When signal in single active layer is produced by only small number of charged tracks, Landau fluctuations matter. This happens for:
  - ✓ Very thin active gaps → broaden the fluctuation of each sample (Landau-type effect)
  - ✓ Very coarse sampling → reduces the number of effective samplings of the shower (sampling effect)
- $a \sim f_{samp}^{-1/2}$ does not represent well resolution anymore
  - ✓ Fluctuations of path length become important, due to predominance of energy lost by Compton/photo-electrons and bigger multiple scattering
  - ✓ When active layer thickness d << λ_{MFP} fluctuations not Gaussian, single track can deposit much more than the mean

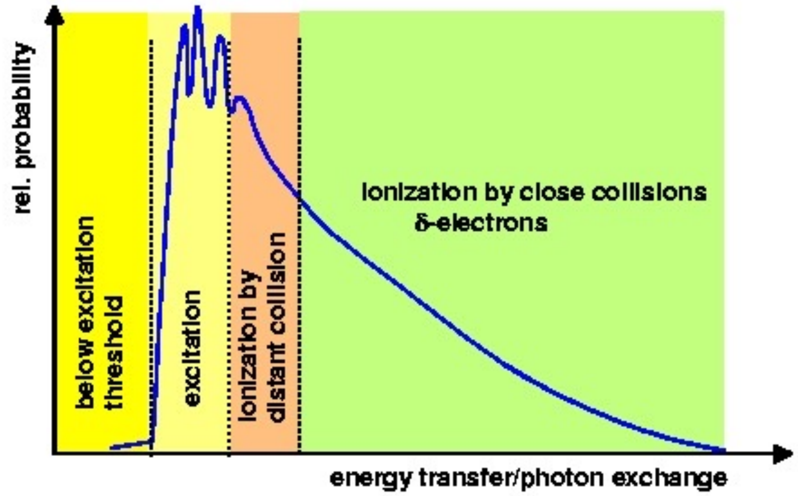


- Silicon detectors usually extremely thin (e.g. 0.3 mm): Landau tail significant if used for calorimetry
- Gas detectors as active medium have very thin gas gaps
  - ✓ Mean energy deposit per gap ~ few keV → very poor sampling
  - ✓ Compensated by large number of gaps and/or gas amplification

Landau fluctuations impact on sampling calorimeters

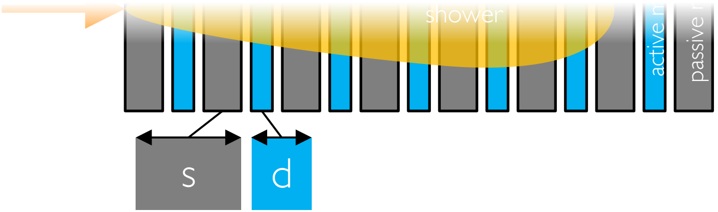
- Recap from Lecture 2: Landau distribution describes energy loss in thin slab of material
  - ✓ Asymmetric: long high-energy tail from rare delta-ray production
  - ✓ Most Probable Value (MPV) << mean; sigma comparable to MPV





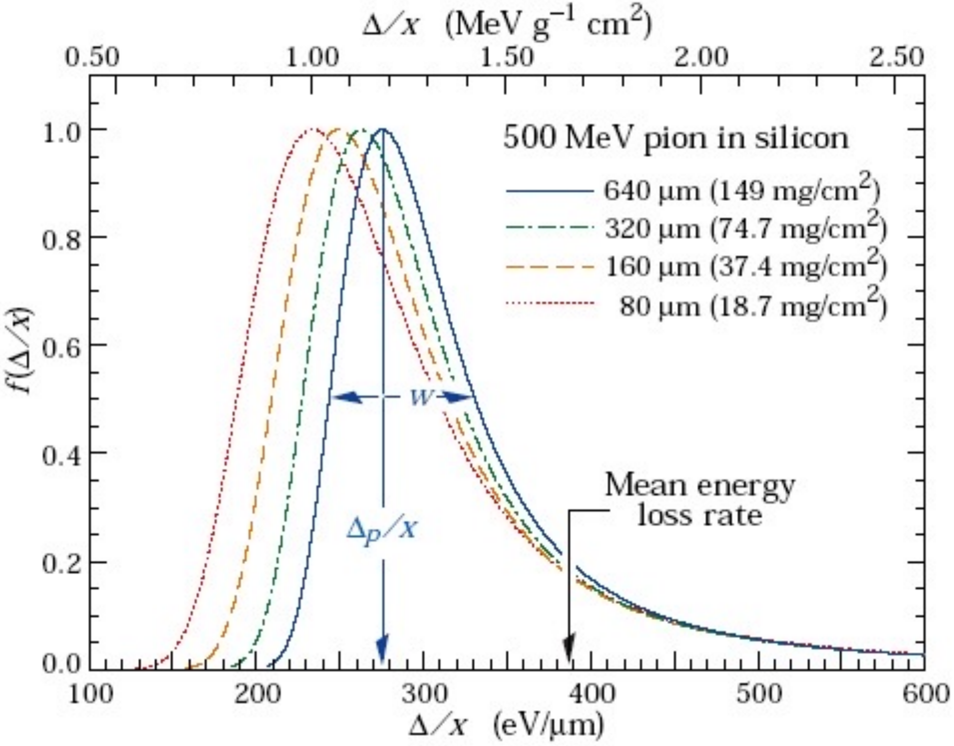
- **When do Landau fluctuations matter for sampling calorimeters?**
  - ✓ Small *d* makes the energy deposited per active sample less Gaussian
  - ✓ When the active layer is thin enough that signal in one sample is produced only by small number of charged tracks
    - Typical examples: thin Si layers or thin gas gaps
- **Direct impact on resolution: adds *non-Gaussian* contribution to stochastic term!**
  - ✓ Correction factor to effective formula for thin active layers → a_{samp} increases by ~20-40%
  - ✓ Particularly relevant for: e.g Si-W sampling calorimeters (CMS HGCAL)
  - ✓ Landau correction can be built into energy reconstruction algorithm

When non-Gaussian single-sample fluctuations matter



- Active layer impact rule of thumbs
  - ✓ $d/X_0 > 1$: Gaussian approximation adequate
  - ✓ $d/X_0 < 0.3$: Landau broadening increases σ_E/E

- Example: Si active layer d ~ 0.3 mm
  - ✓ d ~ 0.3 mm → $d/X_0 = 0.0032$
  - ✓ Signal in one layer comes from very small path length
  - ✓ Single-layer deposits are strongly non-Gaussian
  - ✓ Landau tails can noticeably broaden the resolution



Stochastic term "effective" parameterization

- Sampling calorimeter resolution formula for homogeneous active medium

$$a[\%/\sqrt{GeV}] \sim 5.5\sqrt{e_{\mathrm{sampling}}[\mathrm{MeV}]}$$

  - ✓ $e_{sampling}$ = average energy deposited by MIP in one active layer [MeV]
  - ✓ Consequence: thinner active layers → smaller $e_{sampling}$ → better resolution

- A few values… *

* check the formula for the various case, see whether it works…

Detector	Active	Absorber	a	Use
ATLAS LAr EM	LAr 2 mm †	Pb	10%	EM
ATLAS TileCal	scintillator 3mm ‡	Fe	52%	Hadronic
CMS HCAL	scintillator 3.7 mm	Brass	65%	Hadronic

† d/X_0 = 0.2/14 ~ 0.014 for a 2 mm LAr gap (X_0 ~ 14 cm), $(dE/dx)|_{MIP}$ for LAr ~ 1.5 MeV/cm: ATLAS LAr is well into the Landau-dominated regime!

‡ see exercise in next slides!

Exercise: compute *a* for ATLAS TileCal

Exercise: compute *a* for ATLAS TileCal using effective parameterization formula

$$a[\%/\sqrt{GeV}] \sim 5.5\sqrt{e_{\text{sampling}}[\text{MeV}]}$$

Exercise: compute *a* for ATLAS TileCal

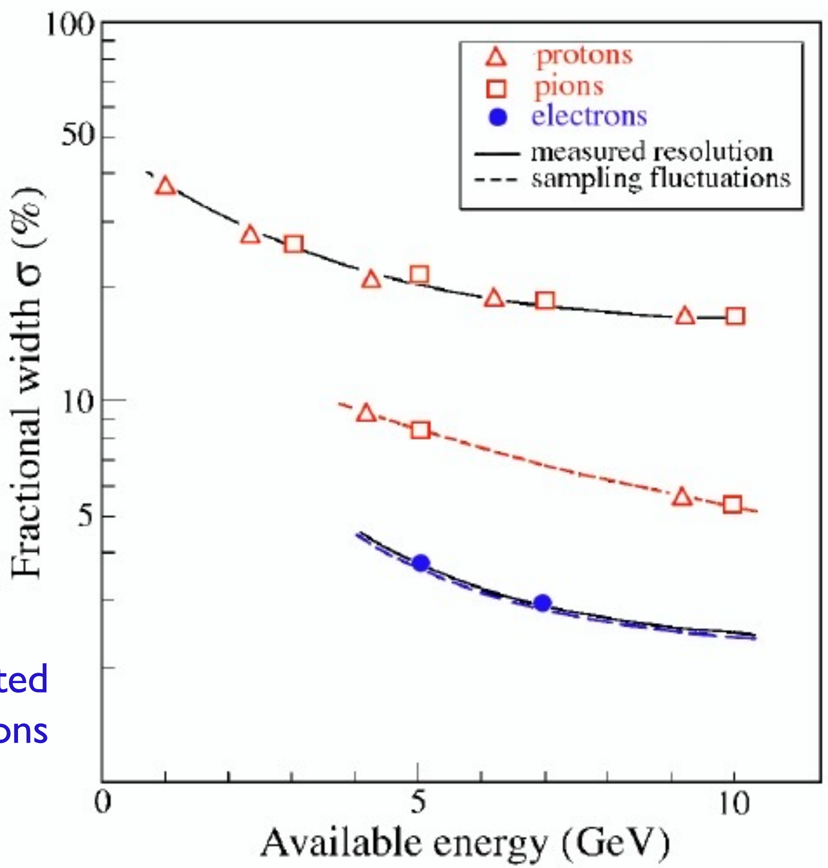
Exercise: compute *a* for ATLAS TileCal using effective parameterization formula

$$a[\%/\sqrt{GeV}] \sim 5.5\sqrt{e_{\mathrm{sampling}}[\mathrm{MeV}]}$$

1. Compute $\varepsilon_{sampling}$ = MIP energy deposited in one active scintillator layer (d = 3 mm)
   - ✓ Polystyrene scintillator: $(dE/dx)|_{MIP} \sim 2.0$ MeV/cm
   - ✓ scintillator layer thickness d = 3 mm = 0.3 cm
   - ✓ $\varepsilon_{sampling} = (dE/dx)|_{MIP} \times d = 2.0$ MeV/cm $\times$ 0.3 cm = 0.60 MeV
2. Apply effective parameterization for sampling-term contribution
   - ✓ a_{samp} = 5.5 × sqrt(0.60) ~ 5.5 × 0.775 ~ 4.3 %/sqrt(GeV) *(sampling fluctuations only)*
3. Compare with measured stochastic term (from table in previous slide)
   - ✓ a_{meas}(TileCal, hadronic) ~ 52 %/sqrt(GeV)
   - ✓ a_{meas} / a_{samp} = 52 / 4.3 ~ 12× (!)

Sampling fluctuations alone do NOT explain hadronic resolution! *Dominant effect: non-compensation (e/h != 1)*
→ fluctuating EM fraction in hadronic showers
→ large intrinsic stochastic contribution

Sampling fluctuations in EM and hadronic showers



Sampling calorimeter
1.5 mm Iron + 2 mm LAr

EM resolution dominated by sampling fluctuations

Sampling fluctuations only minor contribution to hadronic resolution in non-compensating calorimeter!

4.4

Noise Term, Constant Term & Leakage

Noise term: electronic noise

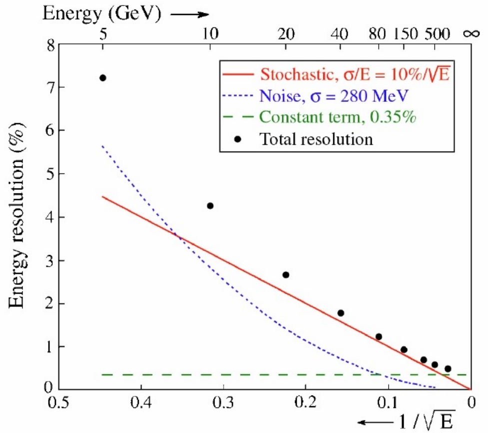
$$\frac{b}{E}$$

- Detector signal = current pulse (directly or after conversion), energy ~ to collected charge
- Current pulse acquired by readout chain, affected by electronic noise (thermal + amplifier noise)
- Amount of electronics noise depends on detector technique and features of readout circuit
  - ✓ → more details on this in lecture 7
  - ✓ **"Light" calorimeter: noise small** when high-gain multiplication (phototube) in first stage of readout chain
  - ✓ **"Charge" calorimeter: noise larger** due to use of pre-amplifier
    - signal shaping + optimal filtering techniques used to minimize electronic noise
    - Equivalent noise charge:

$$Q = \sqrt{4kTR\delta F}$$

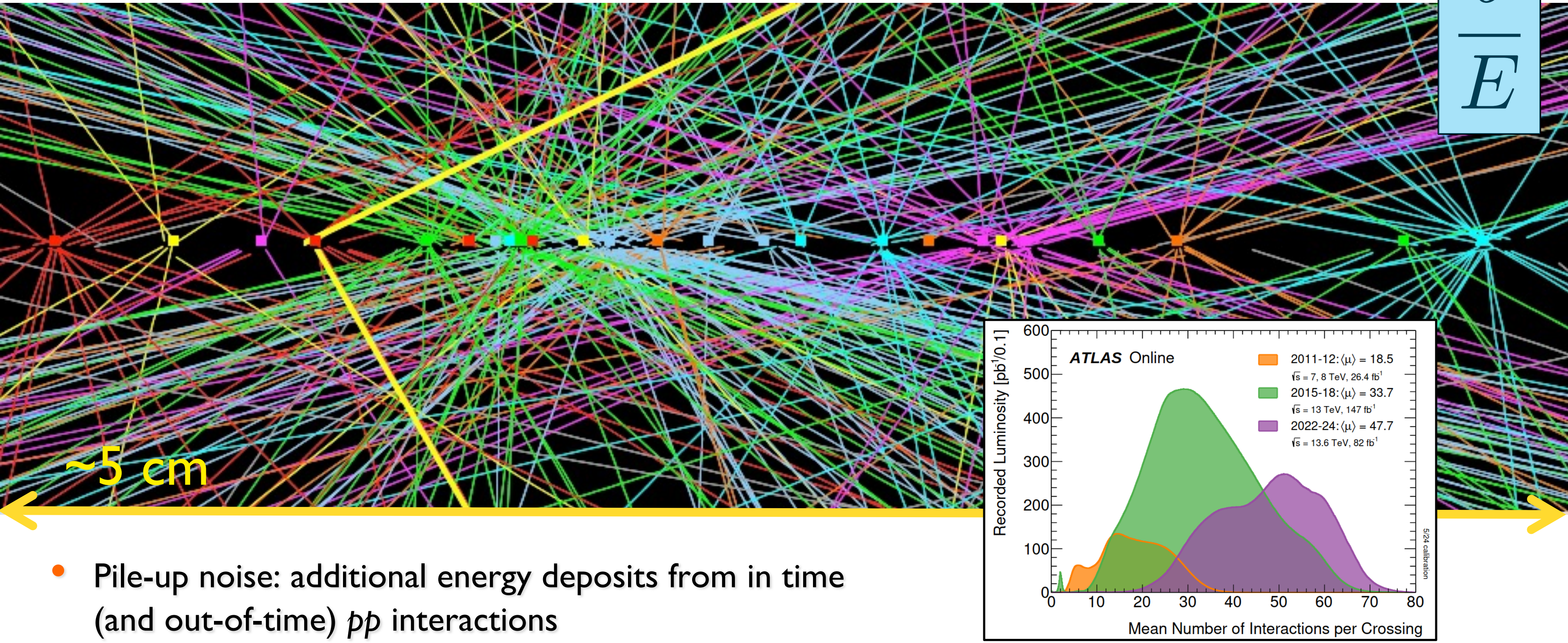
k = Boltzmann constant
T = temperature
R = preamplifier equivalent noise resistance of
dF = preamplifier bandwidth

- *b* depends on amount of considered detector cells (clustering)
  - ✓ cluster size + clustering techniques, e.g. zero suppression (→ more on this in Lecture 7)
- Noise may become dominant below few GeV
  - ✓ noise-equivalent energy is required << 100 MeV for applications in the several GeV region.
- At LHC energies: electronic noise usually small (*b* ~ few MeV per cell)
  - ✓ ATLAS LAr: noise ~ 20-50 MeV per cell (depends on gain); cluster noise ~ 200 MeV
  - ✓ CMS ECAL: noise ~ 40 MeV per crystal; cluster noise ~ 200-300 MeV



Noise term: pile-up

$$\frac{b}{E}$$



- Pile-up noise: additional energy deposits from in time (and out-of-time) *pp* interactions
  - ✓ σ_{pileup} ~ 1-2 GeV per cluster/jet area: much larger than electronic noise
  - ✓ HL-LHC: up to 200 simultaneous pp collisions → pile-up noise dominates

The constant term: non-uniformity and calibration limits

c

- Constant term c dominates resolution at high energy
  - ✓ Example: CMS ECAL a ~ 3%, c ~ 0.5%
    - → constant term dominates above ~25 GeV

$$E > \left(\frac{a}{b}\right)^2 \text{GeV}$$

- Sources of the constant term
  - ✓ **Inter-channel response non-uniformity** (after calibration): ~0.3-1%
    - e.g. light yield non-uniformity; mechanical deformations; local non-hermeticity
  - ✓ **Longitudinal leakage** fluctuations: ~0.1-0.3% (for well-designed calorimeter)
  - ✓ **Temperature gradients**: ~0.2-0.5% per degree C for $PbWO_4$ (~ -2%/C light yield)
  - ✓ **Dead material in front of calorimeter**
    - significant if not corrected by pre-sampler or other calibration techniques

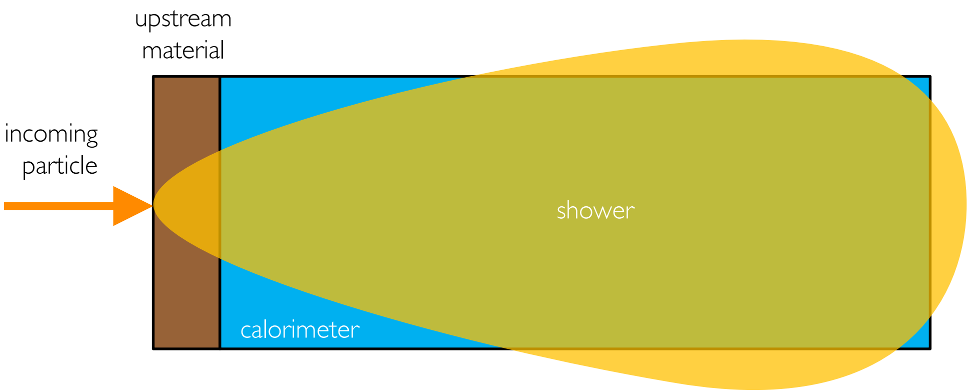
- Minimizing the constant term: the calibration challenge!
  - ✓ Mechanical control
  - ✓ Temperature control: e.g. CMS ECAL stabilized to 0.1 C
  - ✓ Inter-channel calibration: do all you can do with first-principle knowledge + use "standard candles" events (i.e. $\pi^0 \to \gamma\gamma$, $Z \to ee$ (more on this in lecture 7)

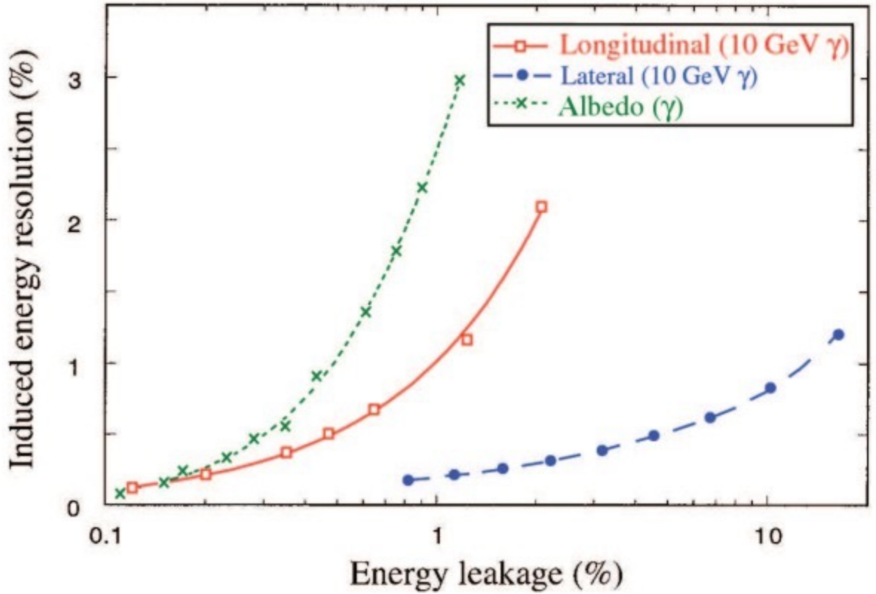
Leakage

- Due to calorimeter finite size, shower might not be not fully contained
  - ✓ Longitudinal leakage
  - ✓ Lateral leakage
  - ✓ Albedo (= backward leakage through front of calorimeter)
  - ✓ Upstream energy losses
    - → more on pre-sampler and correction algorithms in Lecture 7
  - ✓ Non-hermetic coverage
- Dominate at high energies for homogeneous calorimeters since stochastic term very small
- Lateral leakage: limited influence
- Longitudinal leakage: strong influence!
  - ✓ Fluctuates event-by-event → adds to c term
- Impact on resolution (possible parameterization):

$$\frac{\sigma_E}{E} = \left(\frac{\sigma_E}{E}\right)_{f=0} [1 + 2f\sqrt{E}]$$

f = average fraction of shower leakage





Energy resolutions example

Technology (Experiment)	Depth	Energy resolution	Date	
NaI(Tl) (Crystal Ball)	$20X_0$	$2.7\%/\mathrm{E}^{1/4}$	1983	Homogeneous
$Bi_4Ge_3O_{12}$ (BGO) (L3)	$22X_0$	$2\%/\sqrt{E} \oplus 0.7\%$	1993	Homogeneous
CsI (KTeV)	$27X_0$	$2\%/\sqrt{E} \oplus 0.45\%$	1996	Homogeneous
CsI(Tl) (BaBar)	$16–18X_0$	$2.3\%/E^{1/4} \oplus 1.4\%$	1999	Homogeneous
CsI(Tl) (BELLE)	$16X_0$	1.7% for $E_\gamma > 3.5$ GeV	1998	Homogeneous
$PbWO_4$ (PWO) (CMS)	$25X_0$	$3\%/\sqrt{E} \oplus 0.5\% \oplus 0.2/E$	1997	Homogeneous
Lead glass (OPAL)	$20.5X_0$	$5\%/\sqrt{E}$	1990	Homogeneous
Liquid Kr (NA48)	$27X_0$	$3.2\%/\sqrt{E} \oplus 0.42\% \oplus 0.09/E$	1998	Homogeneous
Scintillator/depleted U (ZEUS)	$20–30X_0$	$18\%/\sqrt{E}$	1988	Sampling
Scintillator/Pb (CDF)	$18X_0$	$13.5\%/\sqrt{E}$	1988	Sampling
Scintillator fiber/Pb spaghetti (KLOE)	$15X_0$	$5.7\%/\sqrt{E} \oplus 0.6\%$	1995	Sampling
Liquid Ar/Pb (NA31)	$27X_0$	$7.5\%/\sqrt{E} \oplus 0.5\% \oplus 0.1/E$	1988	Sampling
Liquid Ar/Pb (SLD)	$21X_0$	$8\%/\sqrt{E}$	1993	Sampling
Liquid Ar/Pb (H1)	$20–30X_0$	$12\%/\sqrt{E} \oplus 1\%$	1998	Sampling
Liquid Ar/depl. U (DØ)	$20.5X_0$	$16\%/\sqrt{E} \oplus 0.3\% \oplus 0.3/E$	1993	Sampling
Liquid Ar/Pb accordion (ATLAS)	$25X_0$	$10\%/\sqrt{E} \oplus 0.4\% \oplus 0.3/E$	1996	Sampling

Exercise: EM energy resolution for H → γγ

Exercise: What are the dominant contributions to the photon energy resolution relevant to H → γγ reconstruction in ATLAS and CMS?

- H → γγ: one of the key Higgs discovery channels
  - ✓ Signal: narrow invariant-mass peak on smooth γγ continuum
  - ✓ Background rejection relies on sharpness of the peak
- Mass resolution requirement
  - ✓ $\sigma_m < 1.5$ GeV → $\sigma_m/m_H \sim 1.2\%$

$$m_{\gamma\gamma} = \sqrt{2E_1^{\gamma}E_2^{\gamma}(1-\cos\theta_{12})}$$

$$\frac{\sigma_{m_{\gamma\gamma}}}{m_{\gamma\gamma}} = \frac{1}{2}\sqrt{\left(\frac{\sigma_{E_1}}{E_1}\right)^2 + \left(\frac{\sigma_{E_2}}{E_2}\right)^2 + \left(\frac{\sigma_{\theta_{12}}}{\tan\theta_{12}}\right)^2}$$

$\theta_{12} \sim 180^\circ$

$\tan\theta_{12} \sim \infty$

Exercise: EM energy resolution for H → γγ

Exercise: What are the dominant contributions to the photon energy resolution relevant to H → γγ reconstruction in ATLAS and CMS?

- Mass resolution requirement
  - ✓ σ_m < 1.5 GeV → σ_m/m_H ~ 1.2%
- For symmetric decay
  - ✓ E_γ ~ $m_H/2$ ~ 60 GeV

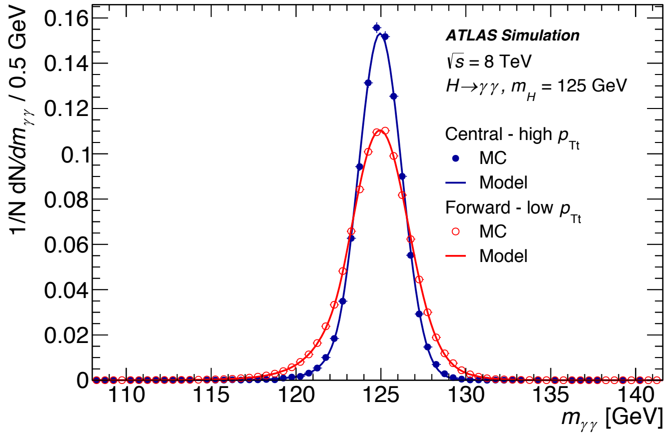
$$\frac{\sigma_{m_{\gamma\gamma}}}{m_{\gamma\gamma}} \simeq \frac{1}{\sqrt{2}} \frac{\sigma_{E_\gamma}}{E_\gamma} \qquad \frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus \frac{b}{E} \oplus c$$

- ATLAS
  - ✓ Stochastic a/√E = 10%/√60 ≈ 1.3%
  - ✓ Noise b/E = 0.3/60 ≈ 0.5%
  - ✓ Constant c ≈ 0.4%
  - ✓ Total ≈ 1.5%
- CMS
  - ✓ Stochastic a/√E = 3%/√60 ≈ 0.4%
  - ✓ Noise b/E = 0.2/60 ≈ 0.3%
  - ✓ Constant c ≈ 0.5%
  - ✓ Total ≈ 0.7%

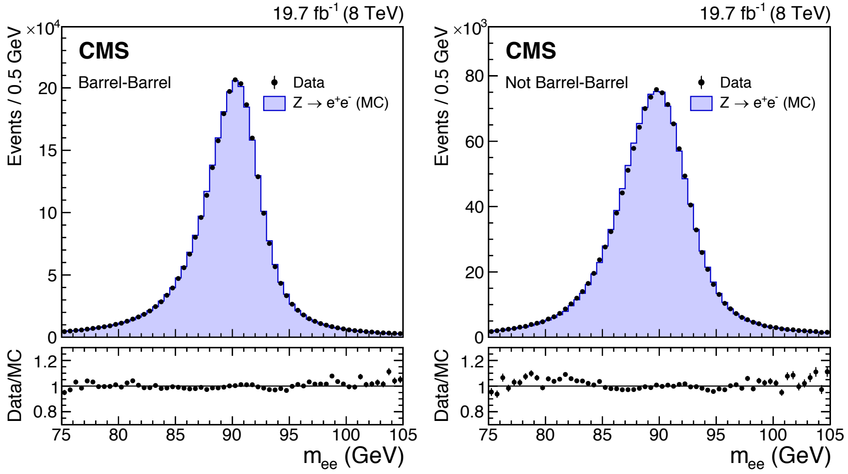
Exercise: EM energy resolution for H → γγ

Exercise: What are the dominant contributions to the photon energy resolution relevant to H → γγ reconstruction in ATLAS and CMS?

- ATLAS Phys. Rev. D. 90, 112015 (2014)



- CMS Eur. Phys. J. C 74 (2014) 3076



- ATLAS mass resolution 1.2-1.7 GeV, depending on category (position in the detector, photon momentum) and data taking conditions (pileup): better than expected (calibration!)

4.5

Hadronic Energy: e/h & Compensation

Callback: the e/h problem

- *Reminder → Lecture 3, Section 3.3*

- **Hadronic shower: two responding components**
  - ✓ EM component ($\pi^0 \to \gamma\gamma$): responds like electrons/photons → response scale e
  - ✓ Non-EM component (spallation, evaporation neutrons): → response scale h

- **key figure of merit: e/h ratio**
  - ✓ e/h ≡ EM response / hadronic response for equal incident energy
  - ✓ e/h = 1: (ideal) "compensating" calorimeter

Typical e/h values

Fe absorber: e/h ~ 1.3
Pb absorber: e/h ~ 1.4
U absorber: e/h ~ 1.05

- **e/h ≠ 1: most real detectors**
  - ✓ 1.1–1.5 typical

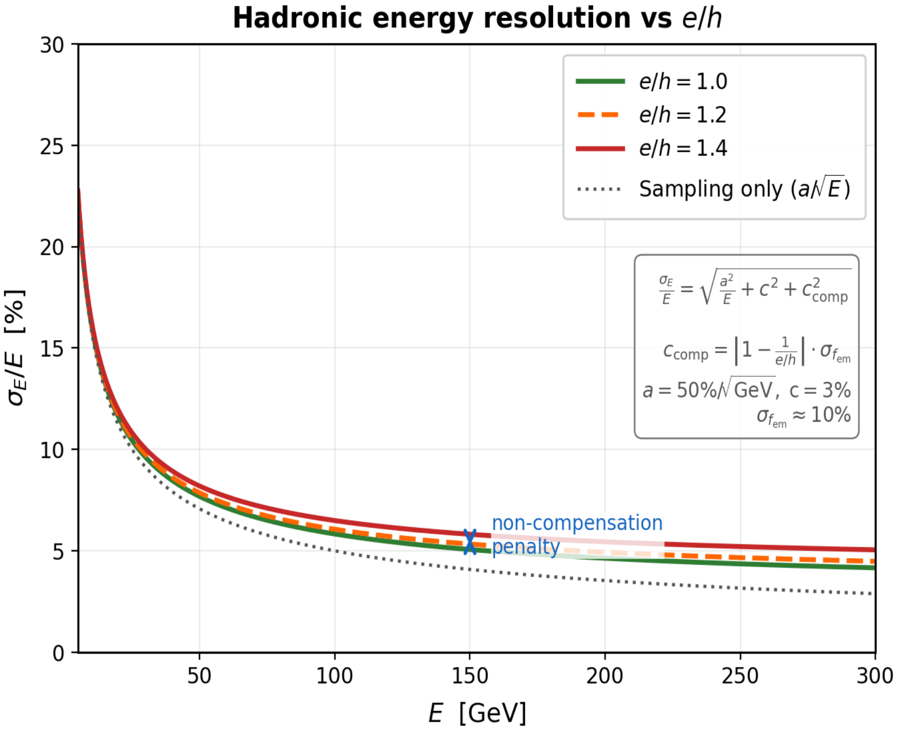
- **Response non-linearity** $$\frac{\langle R_{\mathrm{had}} \rangle}{e} = f_{\mathrm{em}} + (1 - f_{\mathrm{em}}) \left(\frac{e}{h}\right)^{-1}$$
  - ✓ f_{em} increases with energy → $e/\pi(E)$ is energy-dependent

Resolution impact of e/h ≠ 1

$$\frac{\sigma_{\rm comp}}{E} = \left|1 - \left(\frac{e}{h}\right)^{-1}\right| \sigma(f_{\rm em})$$

- **f_{em} fluctuates event-by-event**
  - ✓ $\sigma(fe_m)$ ~ 10–15% at typical energies
  - ✓ With e/h ≠ 1, each event gets a different response bias

- **Result: extra resolution term**
  - ✓ This is a constant-like term
  - ✓ …does not improve as 1/√GeV
  - ✓ e/h = 1.4, $\sigma(f_{em})$ = 10% → σ_{comp} ~ 2.9%



- *Reminder: ATLAS TileCal exercise (Sec 4.3)*
  - ✓ *a_{samp} = 4.3%/√GeV (sampling term alone)*
  - ✓ *$a_{measured}$ ~ 52%/ √GeV → x12 larger!*
  - ✓ *Dominated by e/h ≠ 1 at all practical energies*

Compensation theory

- **Goal: e/h → 1 (compensating calorimeter)**

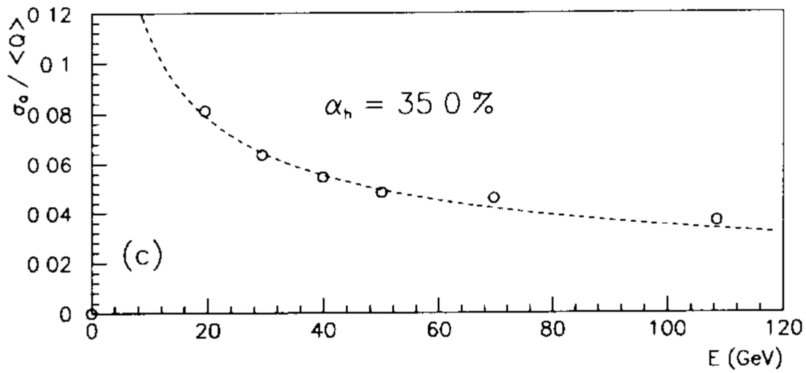
- **Strategy 1: suppress EM response (↓ e)**
  - ✓ High-Z absorber (Pb, U): sub-threshold e^- multiplication suppressed
  - ✓ Fast fission of ^{238}U produces extra hadronic energy → boosts h

- **Strategy 2: recover hadronic signal (↑ h)**
  - ✓ Hydrogen-rich active medium (plastic scintillator)
  - ✓ Evaporation neutrons scatter on ^{1}H → recoil protons → visible signal
  - ✓ Critical: optimal sampling fraction f_{samp} ~ 10–16%

ZEUS compensating calorimeter

Absorber: uranium (^{238}U)
Active: plastic scintillator
e/h = 1.00 ± 0.02
$\sigma/E \sim 35\%/\sqrt{E}$

Proof-of-principle that perfect compensation is achievable



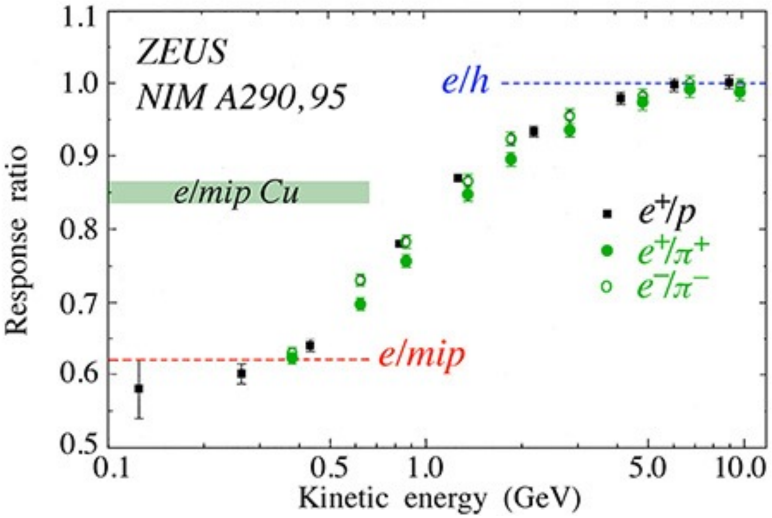
Nucl. Instrum. Meth. A336 (1993) 23–52

The elephant in the (compensating) room

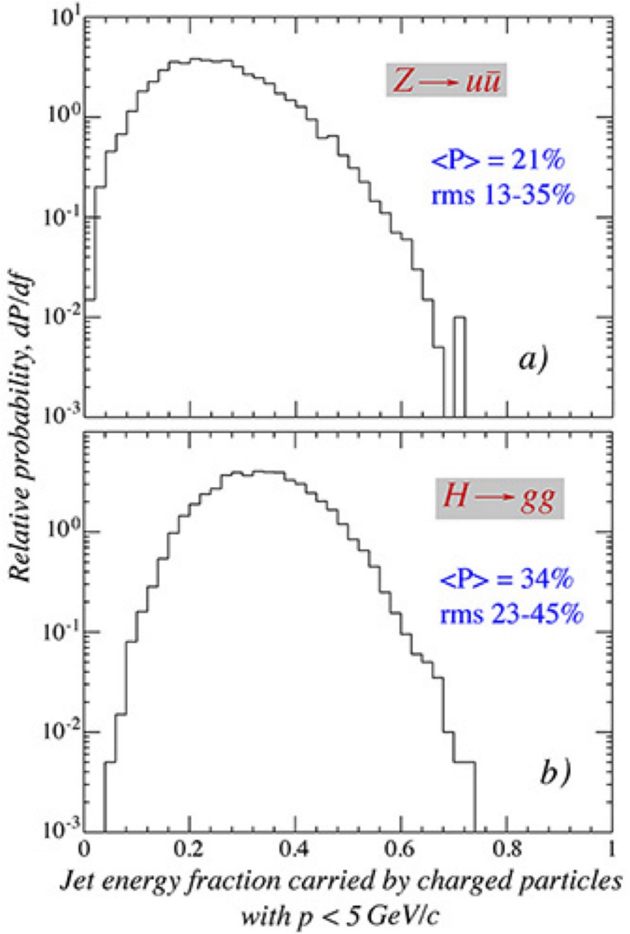
- Compensation is usually achieved with high Z absorber (Pb, U)
  - ✓ Reduce EM response
  - ✓ Generate large number of neutrons
- **Drawback: e/mip gets smaller (~0.6)!**
  - ✓ **Large response non-linearities for low energy hadrons**
  - ✓ **Worse response for jets that for single hadrons!**

(…) As a result of the important contribution from soft jet fragments, and the large event-by-event fluctuations in this contribution, the energy resolution for intermediate vector bosons measured with the compensating ZEUS uranium calorimeter was worse than expected on the basis of the single-pion resolution. Also, the small sam pling fraction required to achieve compensation limited the em energy resolution, to 18%/√E in ZEUS (…)

arXiv:1712.05494

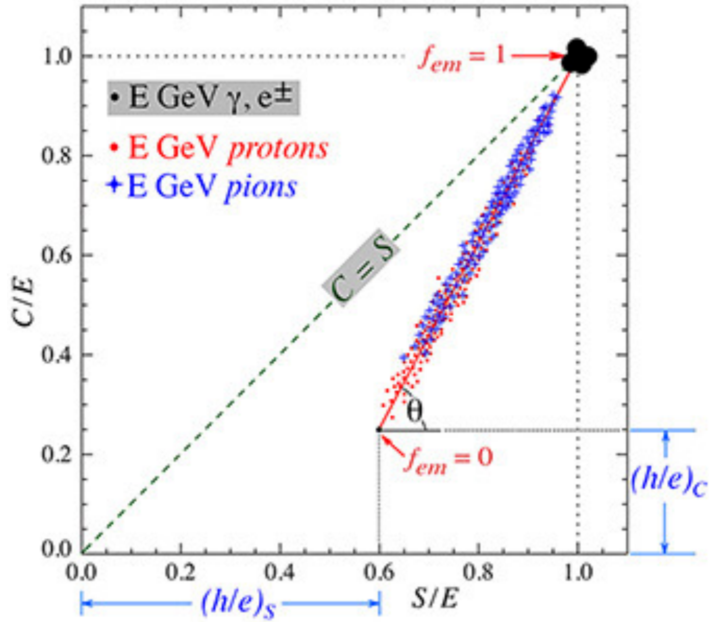


arXiv:1712.05494



Beyond hardware compensation: dual readout calorimeter

- **Idea: measure f_{EM} directly exploiting**
  Cerenkov vs Scintillation light
  - ✓ Electrons in hadronic shower are relativistic down to 200 keV → Cerenkov
  - ✓ Most non-EM energy in hadronic shower from protons from nuclear reaction → Scintillation
- Advantages
  - ✓ No need for high Z absorber (e.g. Cu has e/mip ~0.86)
  - ✓ Can select any sampling fraction
  - ✓ Method does not rely on measuring neutrons (smaller volume, faster integration time)



$$S = E\left[f_{\rm em} + \frac{1}{(e/h)_S}(1 - f_{\rm em})\right]$$

$$C = E\left[f_{\rm em} + \frac{1}{(e/h)_C}(1 - f_{\rm em})\right]$$

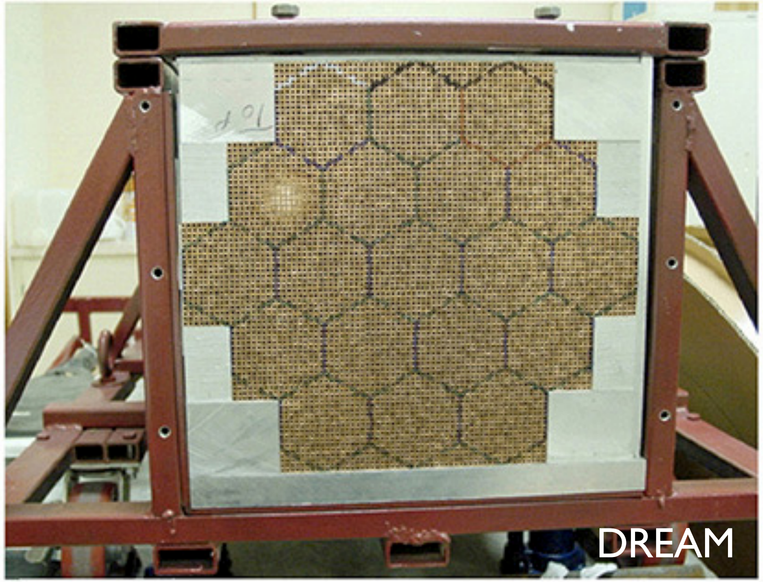
$$f_{\rm em} = \frac{(h/e)_C - (C/S)(h/e)_S}{(C/S)[1-(h/e)_S] - [1-(h/e)_C]}$$

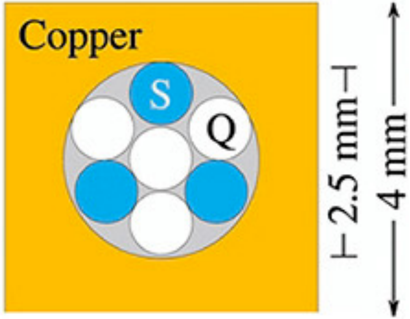
$(e/h)_S$ and $(e/h)_C$ need to be known

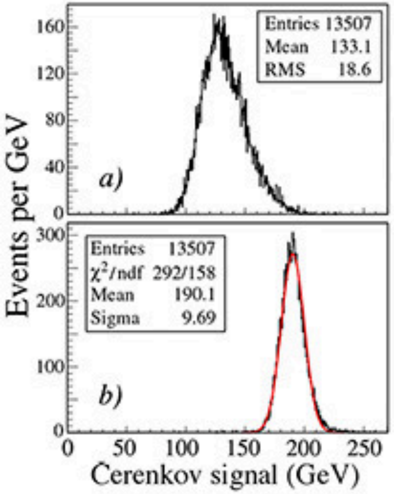
$$\cot\theta = \frac{1-(h/e)_S}{1-(h/e)_C} = \chi$$

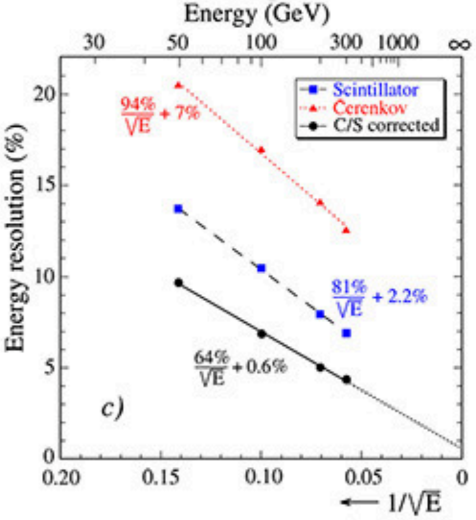
$$E = \frac{S - \chi C}{1 - \chi}$$

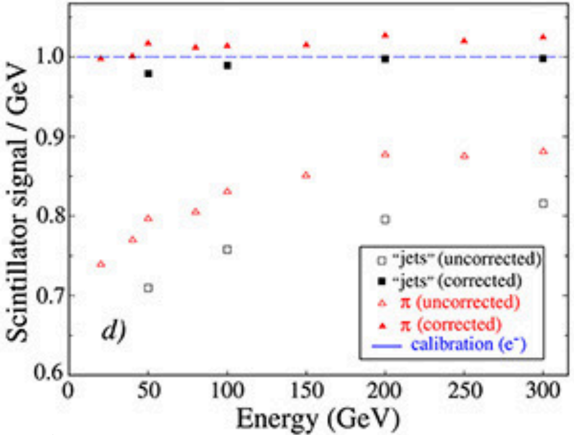
How is a dual readout calorimeter realized? An example











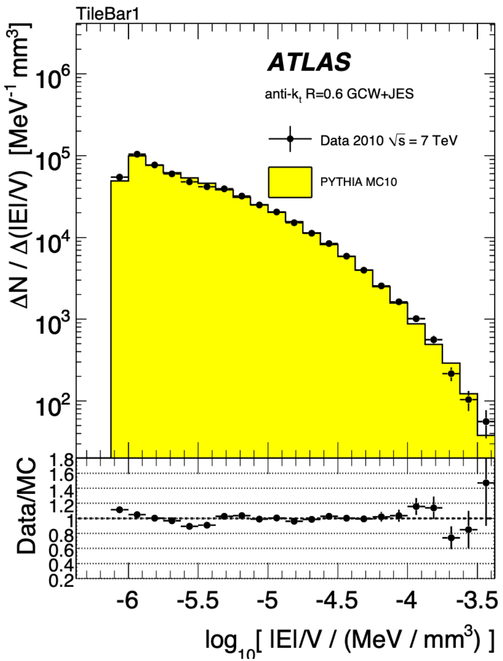
arXiv:1712.05494

Beyond hardware compensation: software compensation

- Software compensation (SWC)
  - ✓ Core idea: weight cells by energy density
  - ✓ Weight high-density cells (likely EM) with

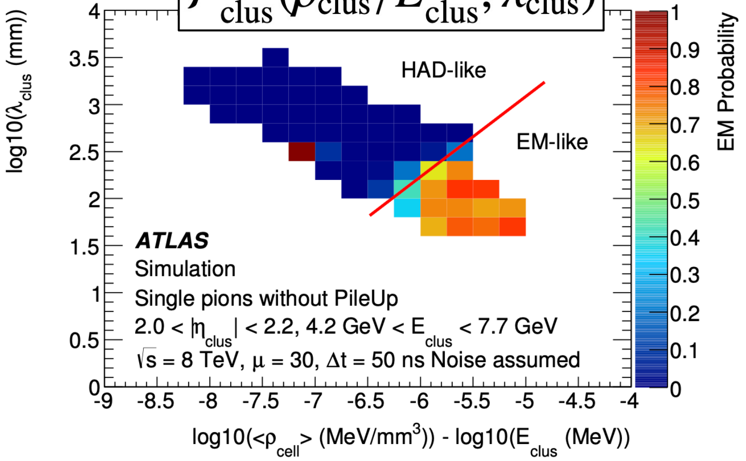
$$w_{\text{cell}} \sim f\left(\frac{E_{\text{cell}}}{E_{\text{cluster}}}\right)$$

Distribution of cell energy in ATLAS TileCal 1



Eur. Phys. J. C 77 (2017) 490

$$\mathcal{P}^{\text{EM}}_{\text{clus}}(\rho_{\text{clus}}/E^{\text{EM}}_{\text{clus}}, \lambda_{\text{clus}})$$



likelihood for reconstructed topo-clusters to originate from an EM shower as a function of shower depth normalized cluster signal density

Example: ATLAS HCAL SWC: improves hadronic resolution by ~20% at 50 GeV

$$w_{\text{cell}} = \frac{E^{\text{dep}}_{\text{cell}}}{E^{\text{EM}}_{\text{cell}}}$$

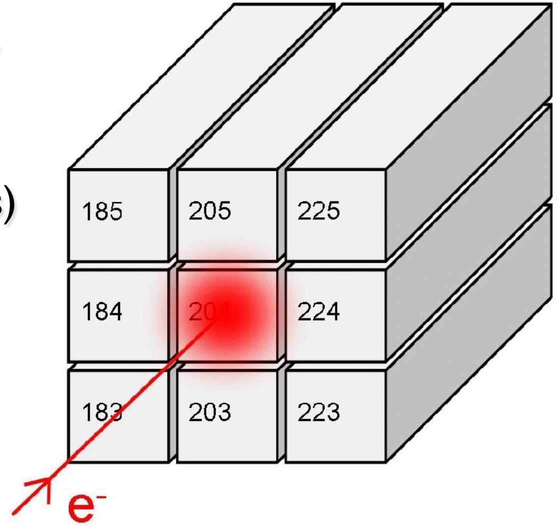
$$w^{\text{cal}}_{\text{cell}} = \mathcal{P}^{\text{EM}}_{\text{clus}} \cdot w^{\text{em-cal}}_{\text{cell}} + (1 - \mathcal{P}^{\text{EM}}_{\text{clus}}) \cdot w^{\text{had-cal}}_{\text{cell}}$$

Or go ever farer, and replace part of the calorimetric measurement with other sources (e.g. tracker) → particle flow

4.6

Position Measurement & Time Resolution

Position measurement in (EM*) calorimeters



- Possible when calorimeter (or readout) is segmented
- Cell size must be comparable to (or smaller than) 1 R_M (Moliere radius)
  - ✓ Reminder: lateral containment → ~90, 95, 99% @ R = 1, 2, 3.5 R_M
  - ✓ Fine segmentation to measure position with good accuracy
- Two algorithms for shower position from cell energies
  - ✓ Simple centroid

$$x_{\text{cog}} = \frac{\sum x_i E_i}{\sum E_i}$$

biased toward cell center

  - ✓ Logarithmic weighting

e.g, in CMS w0 = 5

$$x_{\text{cog}} = \frac{\sum w_i E_i}{\sum w_i} \qquad w_i = w_0 + \ln \frac{E_i}{E_{\text{tot}}}$$

Logarithmic weights correct for the exponential fall-off of EM shower tails

- Physical motivation for log weights: w_i should be proportional to dE/dx

$$E_i \sim e^{-\frac{|x_i - \bar{x}|}{\tau}} \rightarrow w_i \sim -\frac{|x_o - \bar{x}|}{\tau} \sim \ln \frac{E_i}{E_{\text{tot}}}$$

** hadronic position resolution is substantially worse (cell sizes in HCAL usually ~10 cm → σ_x ~ several cm)*

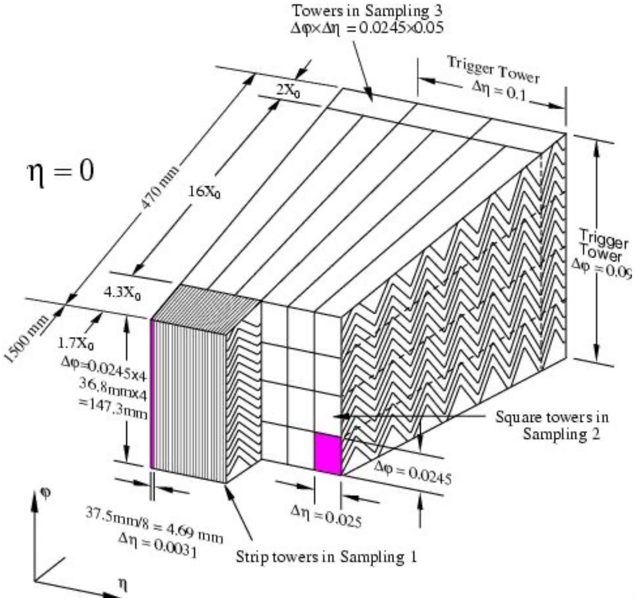
Position measurement in calorimeters

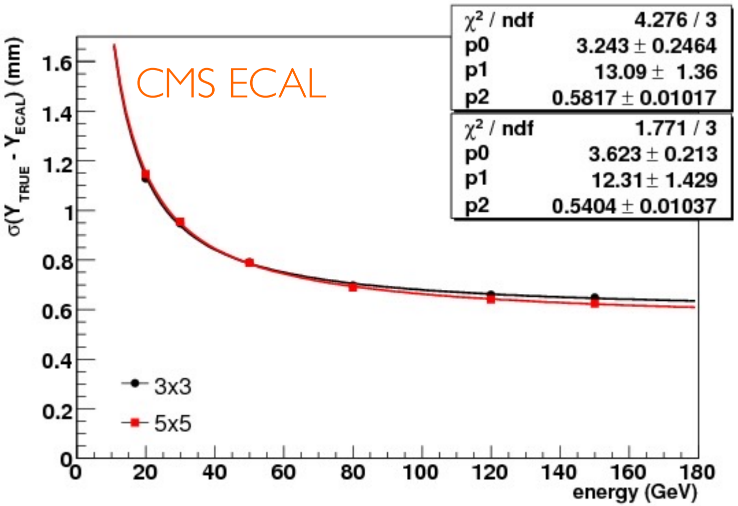
- Position resolution for EM calorimeters

$$\sigma_x \sim \frac{a_{pos}}{\sqrt{(E)}} \oplus c_{pos}$$

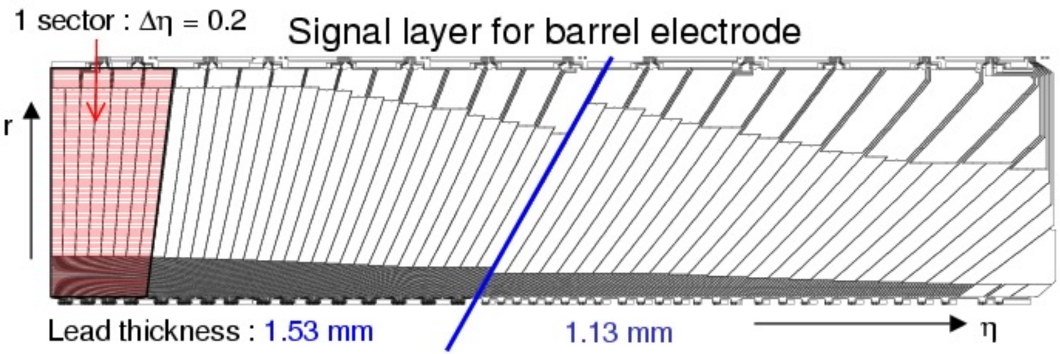
✓ ATLAS LAr : σ_x pos ~ 0.5 mm at 50 GeV (strips layer ~ 4–5 mm @ η~0)

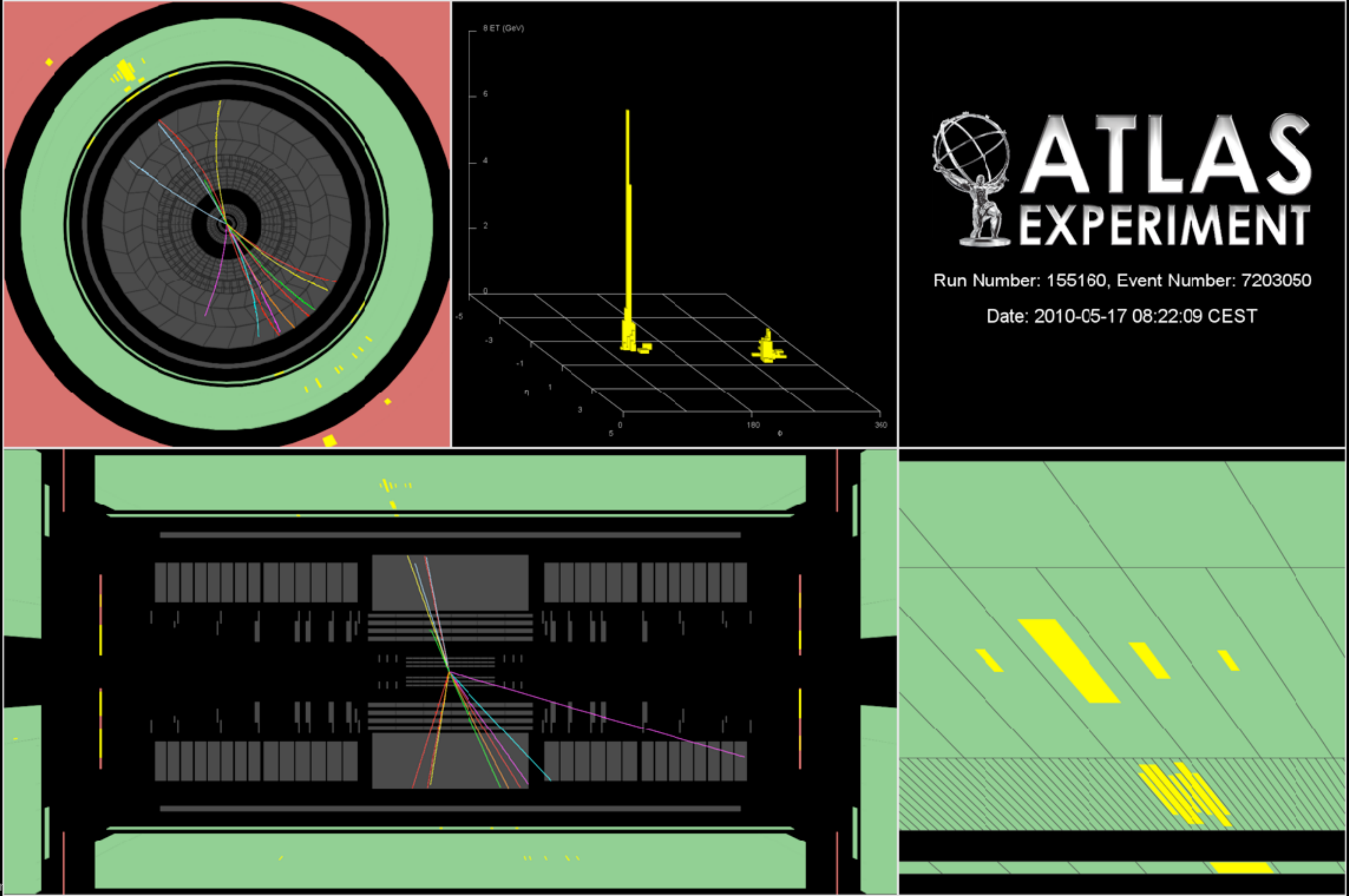
✓ CMS ECAL: cell size ~ 2x2 cm² → uses 5x5 cluster; σ_x~0.7-1.5 mm





ATLAS LAr EMB




ATLAS EXPERIMENT
Run Number: 155160, Event Number: 7203050
Date: 2010-05-17 08:22:09 CEST

Time resolution

- Intrinsic time resolution set by signal rise time / sqrt($N_{photons}$) or signal-to-noise
  - ✓ *Scintillator + PMT/APD: 100 ps – 10 µs*
  - ✓ *Thin silicon detector (10-300 µm): 100 ps – 30 ns*
  - ✓ *Thick (~cm) Si or Ge detector: 1-10 µs*
  - ✓ *Noble liquid ionization (depends on gap width): ~450 ns*

Type	Speed	Resolution	Cost
Ionization	moderate	moderate	cheap
Crystals	fast	best	expensive
Scintillators	fast	moderate	moderate

- Time resolution depends on both Intrinsic time and signal processing technique
  - ✓ CMS ECAL $PbWO_4$: σ_t < 100 ps at E > 10 GeV (fast crystal + fast APD)
  - ✓ ATLAS LAr: limited by 450 ns drift time → σ_t ~ ns (electronics-limited)

- 4D calorimetry: adding precision timing to calorimeter measurements
  - ✓ CMS MTD (Minimum Ionizing Particle Timing Detector): 30-50 ps per track
  - ✓ CMS HGCAL: < 50 ps per hit (LGAD sensors) → pile-up rejection from timing

What did we learn today?

- **Week 2 (Physics depth)**
  - ✓ Lecture 3: Hadronic shower physics
  - ✓ Lecture 4: Energy resolution from first principles
    - *4.1 From Shower Physics to Calorimeter Design*
      - *Homogeneous → best resolution; sampling trades performance for flexibility and cost.*
    - *4.2 The Three-Term Energy Resolution Formula*
    - *4.3 The Stochastic Term: Sampling Fluctuations*
      - *$\sigma_E/E = a/\sqrt{E} \oplus b/E \oplus c$; three independent origins; $a \approx 5.5 \sqrt{e_{sampling}}$ %/√GeV (Wigmans).*
    - *4.4 Noise Term, Constant Term & Leakage*
      - *Noise and pile-up limit low-E resolution; non-uniformity and calibration set the constant floor c.*
    - *4.5 Hadronic Energy: e/h & Compensation*
      - *$e/h \neq 1$ creates constant-like σ_{comp} > sampling term*
      - *compensation (hardware or software) forces $e/h \to 1$*
      - *Hardware compensation works best for single hadrons, for jets not effective, modern alternatives exist*
    - *4.6 Position Measurement & Time Resolution*
      - *Log-weighted centroids reach $\sigma_x \lesssim 1$ mm; timing below 100 ps (PbWO4) enables 4D calorimetry.*