Calorimetry

in particle physics experiments

3.

Hadronic Interactions and Shower Physics

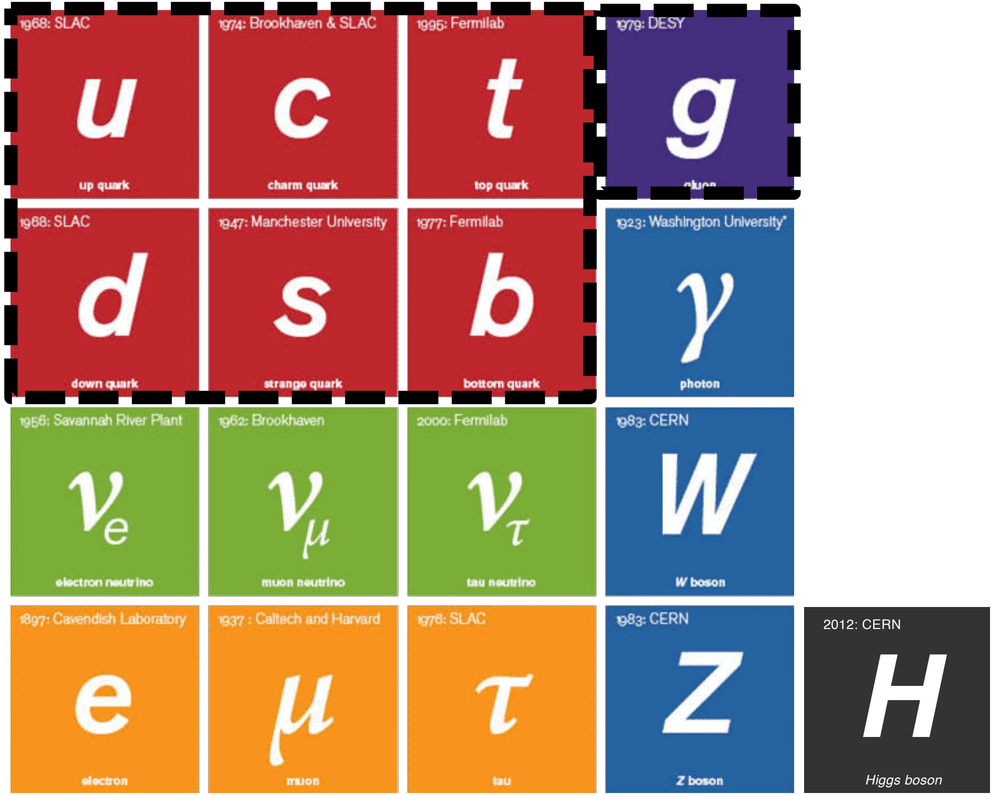
Course roadmap

- **Week 1 (Foundations)**
  - ✓ Lecture 1: Why calorimetry?
  - ✓ Lecture 2: EM shower physics
- **Week 2 (Physics depth)**
  - ✓ Lecture 3: Hadronic shower physics
  - ✓ Lecture 4: Energy resolution from first principles
- **Week 3 (Technology)**
  - ✓ Lecture 5: Calorimeter Technologies (real-life EM and Hadronic calorimeters)
  - ✓ Lecture 6: Calorimeter Design
- **Week 4 (Systems & Future)**
  - ✓ Lecture 7: Signal chain, readout, calibration
  - ✓ Lecture 8: Future calorimetry

What particle do we measure with calorimeters?

hadronic calorimeter

(quarks and gluons form hadron jets)



Today's Lecture

- **Week 2 (Physics depth)**
  - ✓ Lecture 3: Hadronic shower physics
    - *3.1 Nuclear Interactions and Interaction Length*
    - *3.2 Hadronic Shower Structure & Development*
    - *3.3 Hadronic Response & the e/h ratio*
    - *3.4 Shower Time Evolution and Signal Shaping*
    - *3.5 Bridge: What Would a Perfect HCAL Need?*

  - ✓ Lecture 4: Energy resolution from first principles

3.1

Nuclear Interactions and Interaction Length

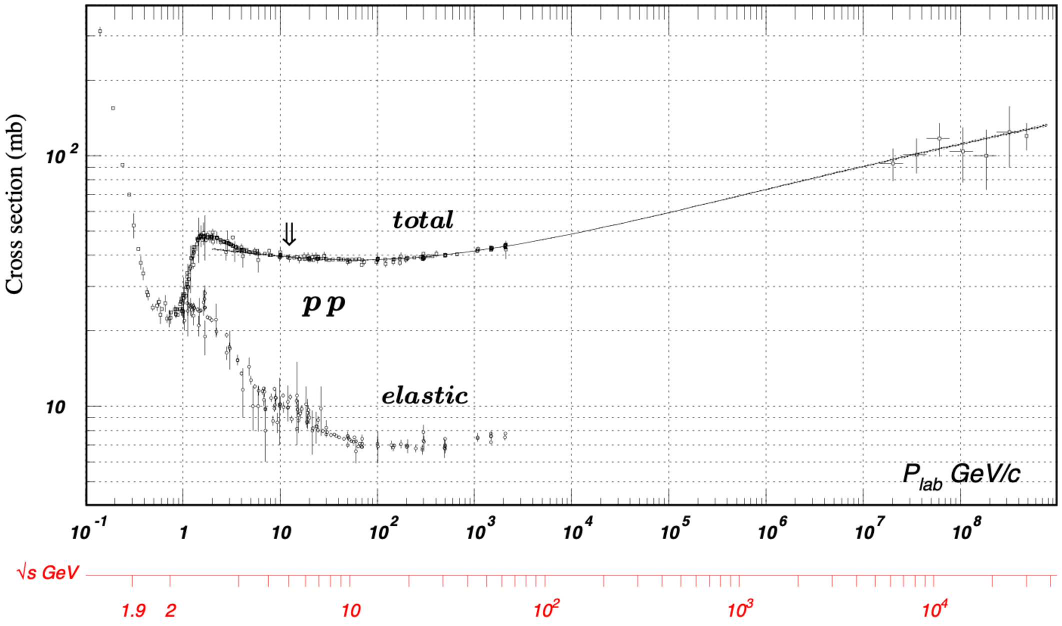
Interaction of hadrons with matter

- Hadrons…
  - ✓ … are bound state of quarks and gluons (partons) → strong interaction
  - ✓ … can be electrically charged → EM interaction
  - ✓ … interact with matter giving rise to *hadronic showers* driven by both strong and electromagnetic interactions

- Hadron-matter interaction is a complex process: mostly dealing with inelastic interactions where secondaries can (will) also be strongly-interacting particles

- When a hadron strikes a matter nucleus…
  - ✓ Interaction between partons (via strong force)
  - ✓ Excitation and break-up of the nucleus
    - nucleus fragmentation/hadronization/production of secondary particles
      - Charged hadrons: $\pi^{\pm}$, K, p, …
      - Charged leptons: $\mu^{\pm}$, $e^{\pm}$
      - Neutral hadrons: η, π^0, n, …
      - Low energy photons

Hadronic interaction cross-section



$$\sigma_{\rm tot} = \sigma_{\rm el} + \sigma_{\rm inel}$$

- Both with strong energy dependency at low energy
- σ_{el} ~ 10 mb
- σ_{inel} dominant at high energies
  - ✓ Inelastic interaction breaks nucleus apart
  - ✓ σ_{inel} drives hadronic shower development
- Geometric approximation for σ_{inel}:

$$\sigma_{\rm tot}(pA) \sim \sigma_{\rm tot}(pp) A^{2/3}$$

$$\sigma_{\rm inel} \propto \pi r_0^2 (A_1^{1/3} + A_2^{1/3})^2 \propto A^{2/3}$$

For a hadron–nucleus interaction A_1 = 1

Example: pion on nucleus: σ_{inel} ~ 45 $A^{0.7}$ mb

Hadronic interaction length

- Hadronic interaction length λ_{int} = mean free path for *inelastic* hadronic interactions
  - ✓ Note: in principle one should distinguish between collision length and absorption length where the latter consider inelastic interactions only (absorption)

$$\lambda_{\mathrm{int}} = \frac{1}{\sigma_{\mathrm{tot}} n} = \frac{A}{\sigma_{\mathrm{tot}}(pp) A^{2/3} N_A \rho} \propto A^{1/3}$$

$$N = N_0 e^{-\frac{x}{\lambda_{\mathrm{int}}}}$$

$\lambda_{int} \sim 35\ A^{1/3}$ g cm^{-2}

- ✓ Both longitudinal and lateral development of hadronic shower can be (quite approximatively) described by λ_{int}: more on this later…

X_0 vs λ_{int}: comparison for key absorber materials

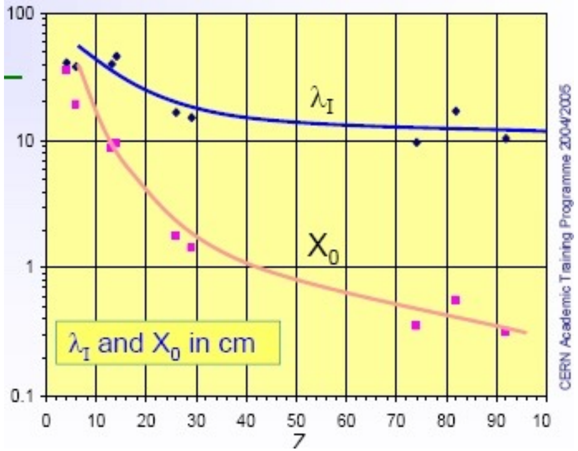
Material	Z	ρ [g/cm³]	X_0 [g/cm²]	X_0 [cm]	λ_{int} [g/cm²]	λ_{int} [cm]	λ_{int} / X_0
Al	13	2.70	24.0	8.89	106.4	39.4	**4.4**
Fe	26	7.87	13.8	1.76	131.9	16.8	**9.5**
Cu	29	8.96	12.9	1.44	134.9	15.1	**10.5**
W	74	19.3	6.8	0.35	185.0	9.6	**27.2**
Pb	82	11.4	6.4	0.56	194.0	17.1	**30.3**
U	92	19.0	6.0	0.32	199.0	10.5	**33.2**

$$X_0 \propto \frac{A}{Z^2}$$

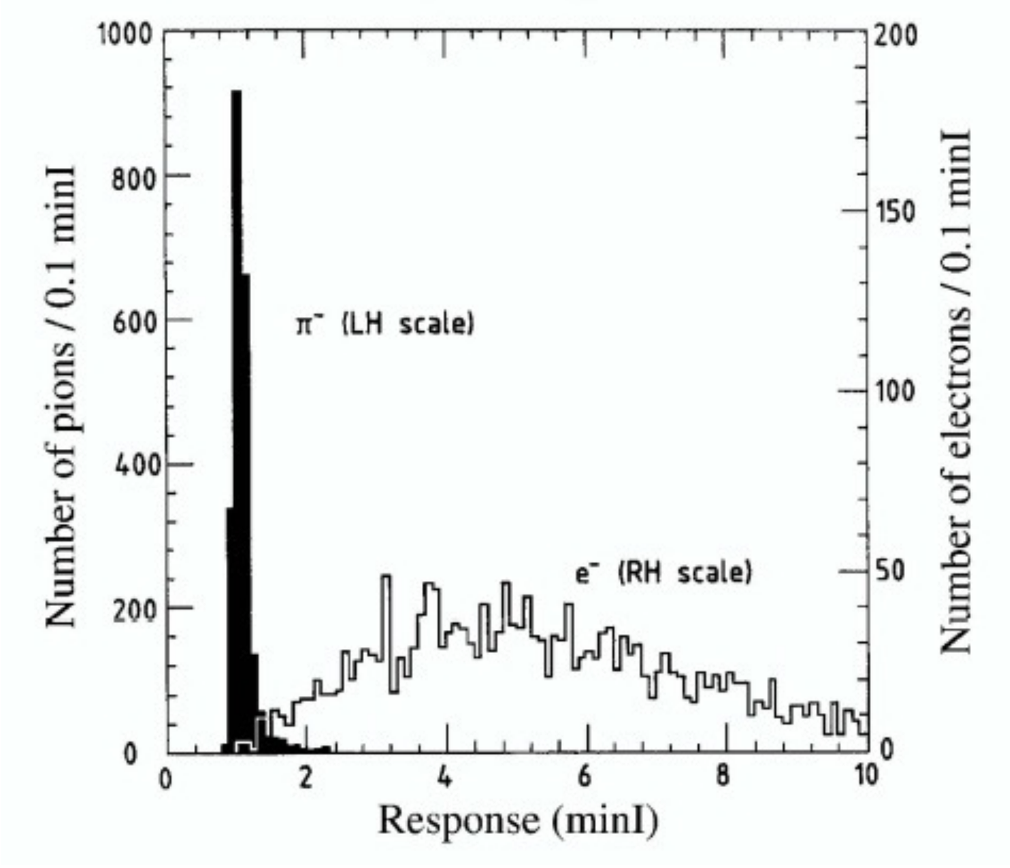
$$\lambda_{\rm int} \propto A^{1/3}$$

$$\frac{\lambda_{\rm int}}{X_0} \propto A^{4/3}$$

- A comparison of two metals
  - ✓ λ_{int}(Cu) = 135 g/cm^2 = 15.1 cm
  - ✓ λ_{int}(Al) = 107 g/cm^2 = 39.7 cm → too light for compact Hadronic Calorimeter
- EM vs hadronic interactions
  - ✓ X_0(Pb) = 0.56 cm vs. λ_{int}(Pb) = 17.1 cm → ratio = 30!
  - ✓ HCAL must be 30x longer than ECAL for same fractional shower containment!
  - ✓ ECAL typically 20-25 X_0 (~14 cm in Pb); HCAL typically 8-11 λ_{int} (~140 cm in Fe)



X_0 vs λ_{int}: useful for particle identification



- λ_{int} /X_0 ratio important for particle identification
- λ_{int} / X_0 ~ 30 in high-Z materials → excellent e / **π** separator
- Example: 1 cm Pb + scintillator plate = “preshower” detector

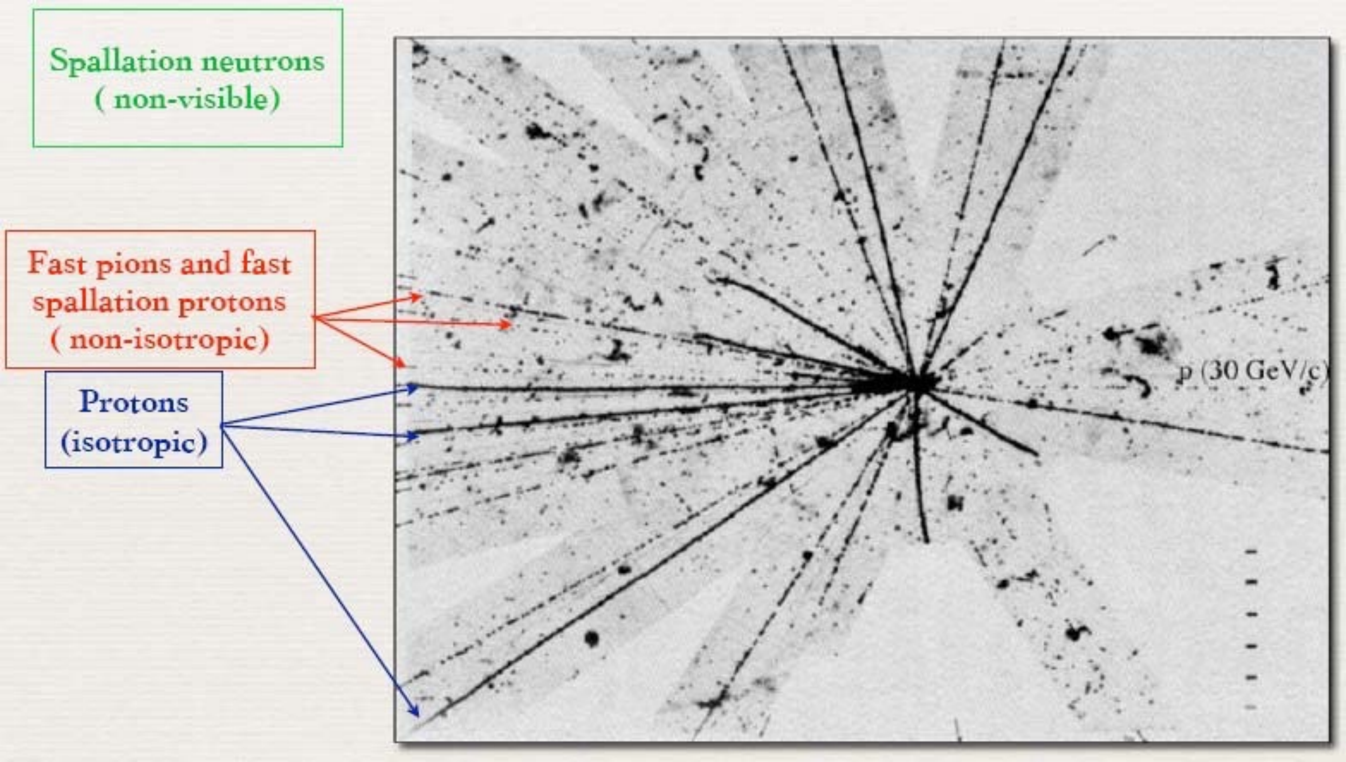
Nuclear processes and invisible energy

- Two classes of effects
  - ✓ Production of energetic secondary hadrons, with mean free path λ_{int}
    - Momenta corresponding to fair fraction of the primary hadron
  - ✓ Significant part of the primary energy consumed in nuclear processes
      - **Excitation**
      - **Nuclear spallation (→ produces slow n)**
      - **Nucleon evaporation**
    - Produce "low" (MeV) energy particles, not all of them are detectable!

- An energy-dependent fraction of the incoming hadron E thus goes in breaking nuclei, in low energy neutrons or undetectable neutrinos → "invisible energy"
  - ✓ Affected by large energy fluctuations → limits energy resolution
    - For example, in lead (Pb):
      - ~42% of hadronic energy: nuclear binding energy (breakup)
      - ~43%: evaporation/recoil below threshold
      - ~12%: escaping neutrinos
      - ~3%: other (muons, charged kaons escaping, etc.)

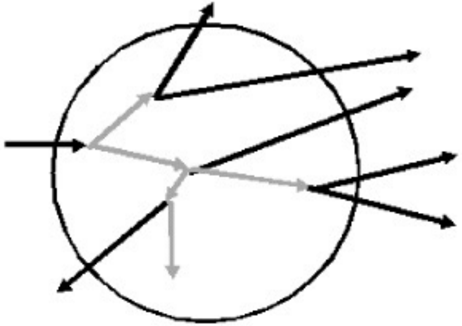
An example of a hadronic interaction…

Nuclear interaction induced by a proton of 30GeV in a photographic emulsion: ~20 ionizing particles produced isotropically, probably all protons, + forward less dense ionization tracks, mostly pions and protons from cascade process

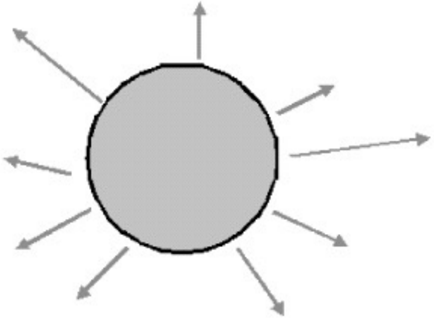


Spallation and soft neutrons

- Spallation process
  - ✓ **Intra-nuclear cascade**
    - Fast hadron traversing the nucleus frees protons and neutrons in number proportional to their numerical presence in the nucleus.
    - Some of these n and p can escape the nucleus
      - For Pb ~1.5 more cascade n than p
    - Nucleons involved in cascade transfer energy to nucleus which gets in excited state
  - ✓ **Nuclear de-excitation**
    - Evaporation of soft (~10 MeV) nucleons and α
    - + fission for some materials
- The number of nucleons released depends on the binding energy
  - ✓ 7.9 MeV in Pb, 8.8 MeV in Fe
- Evaporation mainly releases neutrons
  - ✓ protons are trapped by the Coulomb barrier (12 MeV in Pb, only 5 MeV in Fe)



dominating momentum component along incoming particle direction



isotropic

Can secondary soft neutrons be detected?

- Neutrons (those soft neutrons produced in the “evaporation” stage of nuclear spallation) interaction is based only on strong (and weak) nuclear force
- In order to detect neutrons, they need to convert into charged particles
  - ✓ Possible neutron conversion and elastic reactions
    - $E_n < 20$ MeV
      - $n + {}^6Li \rightarrow \alpha + {}^3H$
      - $n + {}^{10}B \rightarrow \alpha + {}^7Li$
      - $n + {}^3He \rightarrow p + {}^3H$
    - **$E_n < 1$ GeV**
      - **$n + p \rightarrow n + p$**

Slow neutrons can interact with H atoms in active material → their (invisible) energy can be recovered

- In addition
  - ✓ Neutron induced fission $E_n \sim E_{th} \sim 1/40$ eV
  - ✓ Inelastic reactions → hadronic cascades $E_n > 1$ GeV

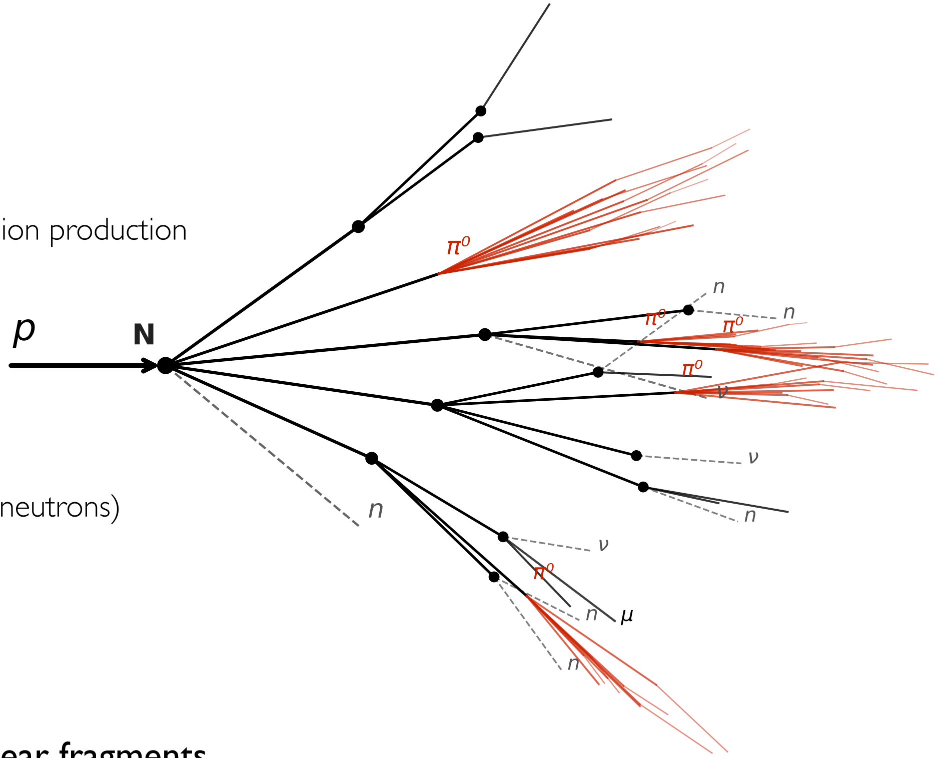
On the other hand, no hope to detect neutrinos in a typical HCAL: interaction cross section ~ 10^{-43} cm^2

3.2

Hadronic Shower Structure & Development

What happens in a hadronic interaction: hadronic shower

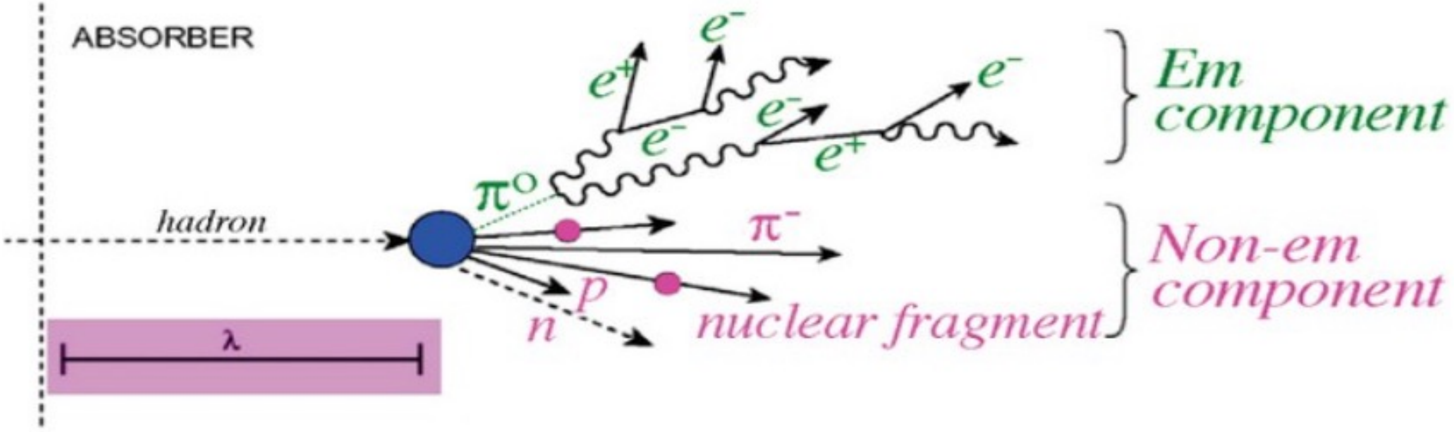
- Hadronic shower development
  - ✓ Hadron + Nucleus → Pions + N* + …
  - ✓ Secondary particles
    - Pions most common secondaries (π^+, π^-, π^0)
    - Undergo further inelastic collisions until they fall below pion production threshold
  - ✓ Sequential decays…
    - $\pi_0 \rightarrow \gamma\gamma$: yields electromagnetic shower
    - Fission fragments → β-decay, γ-decay
    - Neutron capture → fission
    - Spallation (nucleus breaks into fragments + evaporation neutrons)



- Energy partition in hadronic shower (qualitative)
  - ✓ EM component (from π^0 production): ~ 30-50%
  - ✓ Hadronic component: charged pions, protons, nuclear fragments
  - ✓ Invisible energy: nuclear binding energy, slow neutrons, neutrinos

Mean number of secondaries: ~ ln E
Typical transverse momentum: p_T ~ 350 MeV

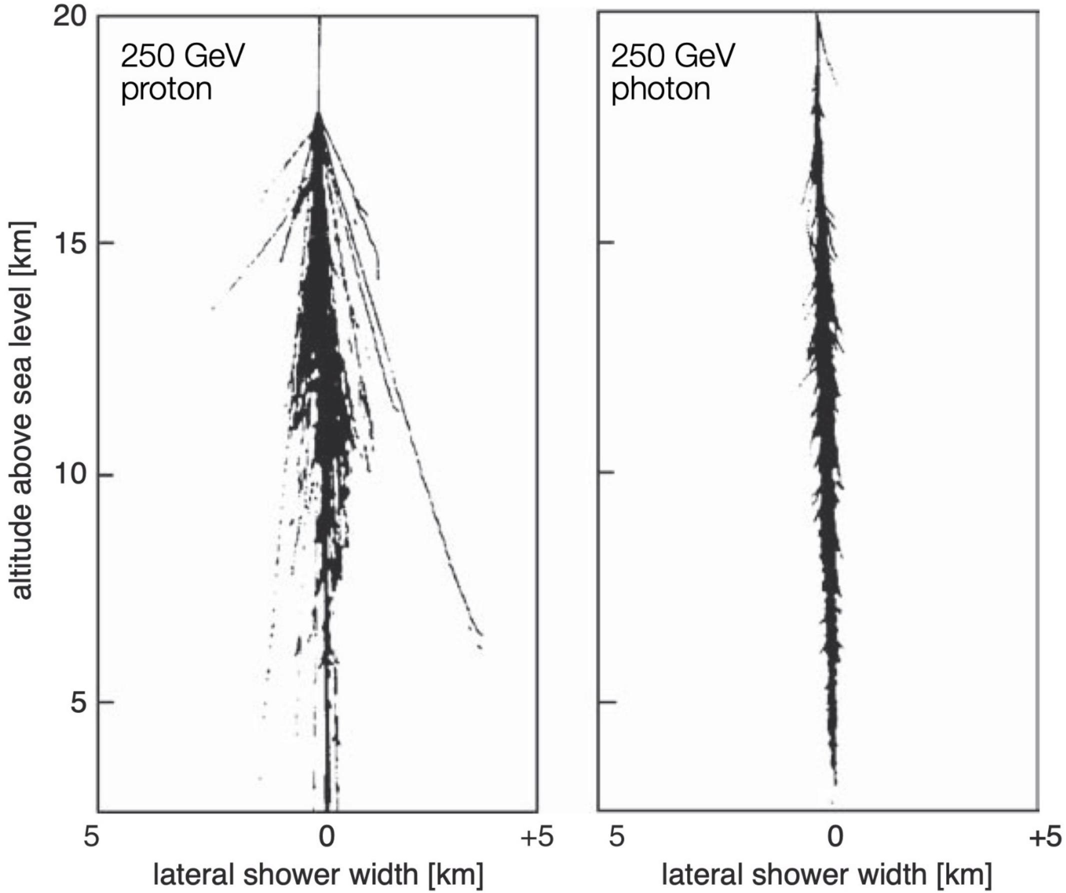
The two-component hadronic shower



- Hadronic shower has 2 distinct *visible* components:
  - ✓ Electromagnetic component: from **π**$_0$ → γγ at each interaction vertex
  - ✓ Hadronic component: charged pions, protons, nuclear fragments, slow neutrons
- f_{em}: fraction of shower energy in the EM component
  - ✓ f_{em} increases with incident energy: **f_{em} ~ ln E** (or also: $1 - (E/E_0)^{k-1}$ [E_0 ~ 1 GeV, k ~ 0.8])
  - ✓ Example: f_{em} ~ 30% at 10 GeV, ~ 50% at 100 GeV, ~ 60% at 1 TeV
- Fluctuation in f_{em} is the dominant contribution to hadronic energy resolution

Hadronic vs. EM showers

Example: high energy proton vs photon in air



Hadronic: $\gtrsim$10× wider laterally, irregular, event-to-event variation large

EM: narrow, self-similar, well-described by X_0

Hadronic shower longitudinal development

- Can parameterize similarly to EM showers…
  - ✓ Longitudinal profile broader and less regular than EM showers
  - ✓ Scale set by λ_{int} $$t = \frac{x}{\lambda_{\text{int}}}$$
  - ✓ Energy at depth t

$$E(t) = \frac{E}{\langle n \rangle^{t}} \qquad E(t_{\max}) = E_{\text{thr}} \qquad E_{\text{thr}} = \frac{E}{\langle n \rangle^{t_{\max}}}$$

$\langle n \rangle \sim 5$–10 $\qquad$ $E_{\text{thr}} \sim 290$ MeV

  - ✓ Shower max

$$\langle n \rangle^{t_{\max}} = \frac{E}{E_{\text{thr}}} \qquad t_{\max} = \frac{\ln(E/E_{\text{thr}})}{\ln\langle n \rangle}$$

$$t_{\max} \sim (0.2 \ln E + 0.7)\lambda_{\text{int}}$$

- But only rough estimate as …
  - ✓ energy sharing between shower particles fluctuates strongly
  - ✓ part of the energy is not detectable (neutrinos, binding energy)
    - partial compensation possible (n-capture & fission)
  - ✓ spatial distribution varies strongly; different range of e.g. $\pi^{\pm}$ and π^0
  - ✓ electromagnetic fraction, i.e. fraction of energy deposited by $\pi^0 \rightarrow \gamma\gamma$ increases with energy

$$f_{\text{em}} \approx f_{\pi^0} \sim \ln E/(1\,\text{GeV})$$

- ***Explanation:*** ***charged hadron contribute to electromagnetic fraction via $\pi^- p \rightarrow \pi^0 n$; the opposite happens only rarely as π^0 travel only 0.2 μm before its decay*** ***("one-way street")***

  - ✓ At energies below 1 GeV hadrons lose their energy via ionization only

- Thus: need Monte Carlo to describe shower development correctly …

Electromagnetic fraction

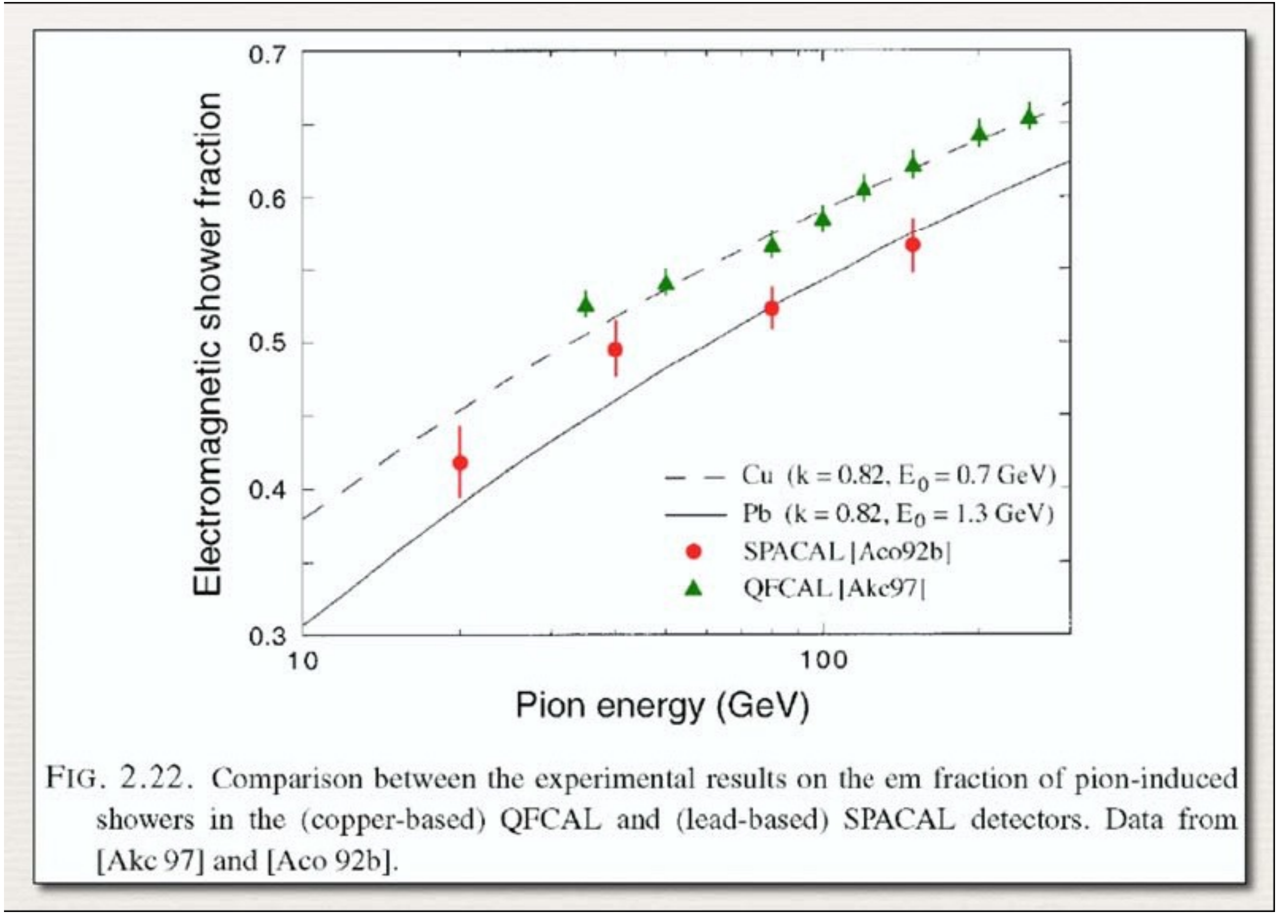
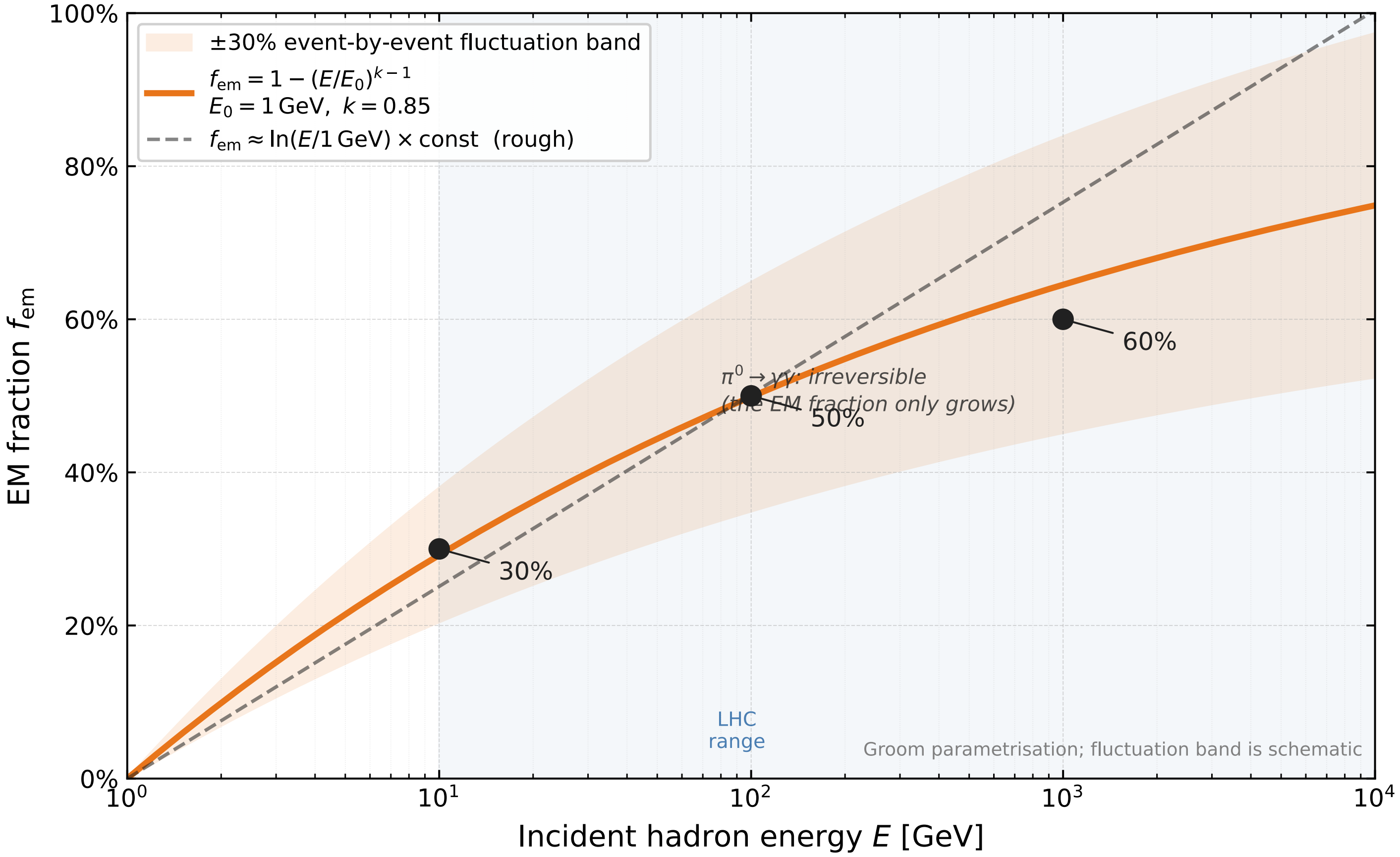


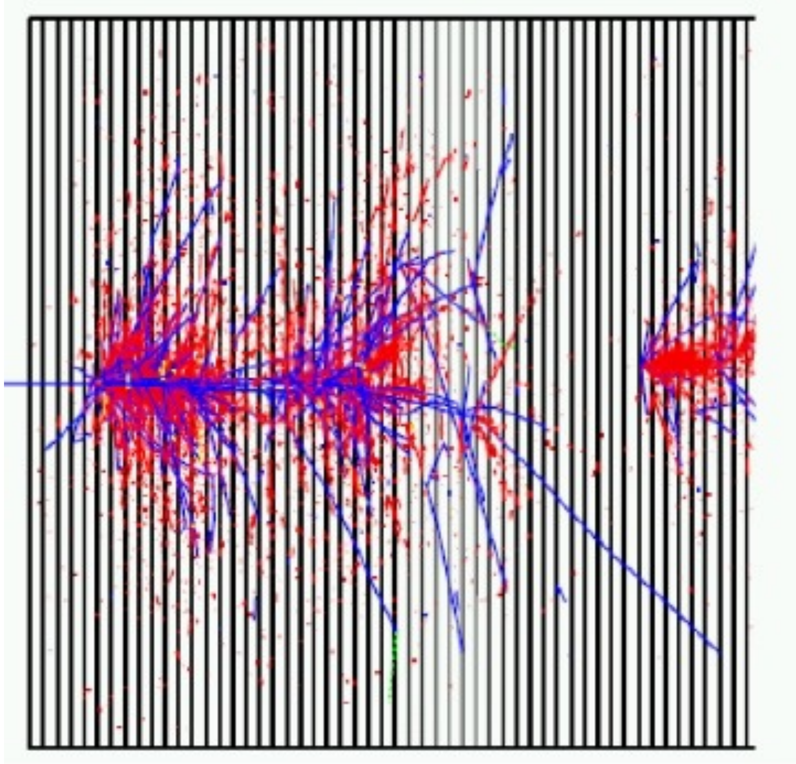
FIG. 2.22. Comparison between the experimental results on the em fraction of pion-induced showers in the (copper-based) QFCAL and (lead-based) SPACAL detectors. Data from [Akc 97] and [Aco 92b].

Electromagnetic fraction



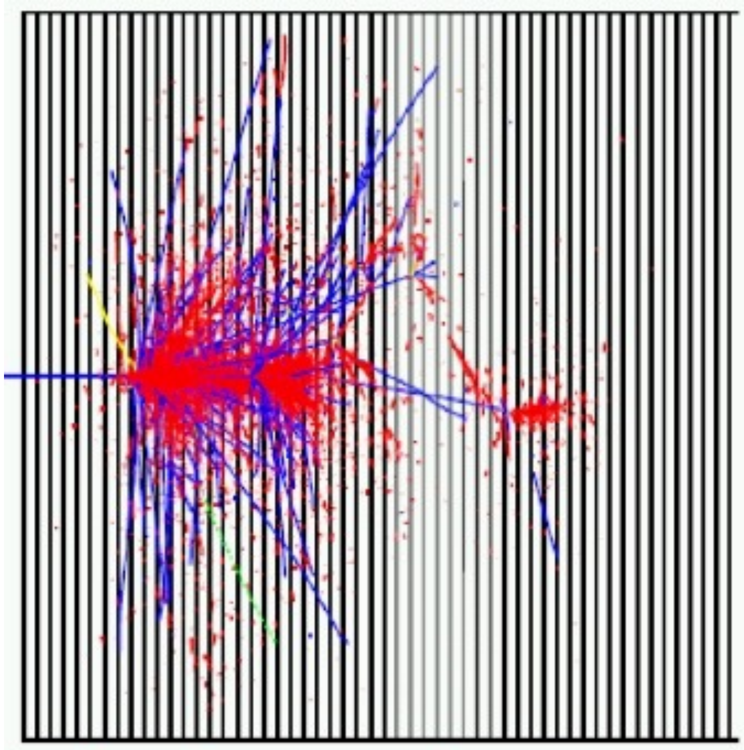
Fluctuations!

Event 1



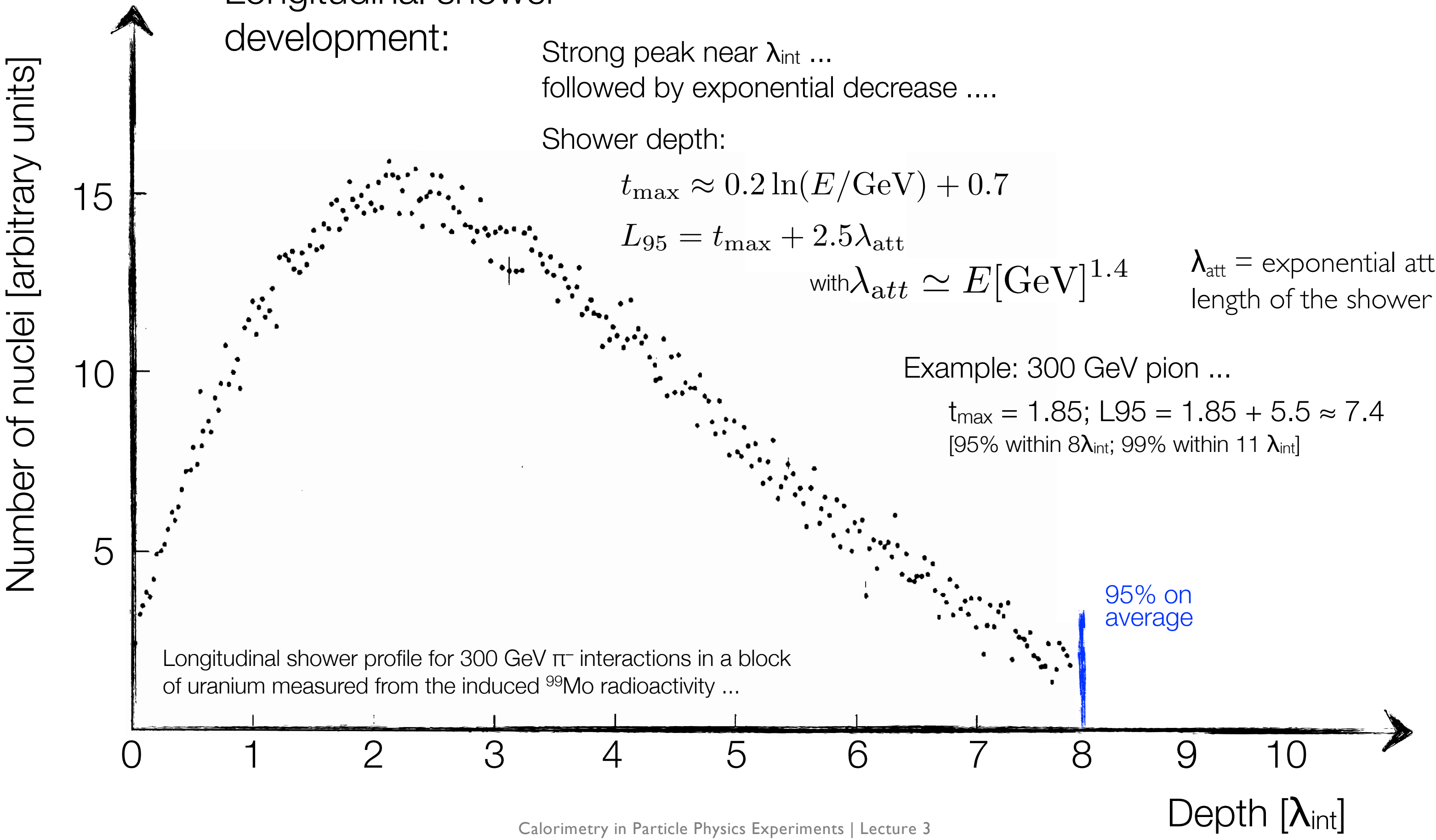
blue = hadronic component

Event 2



red = electromagnetic component

Hadronic shower longitudinal development



Longitudinal shower development:

Strong peak near λ_{int} ...
followed by exponential decrease ....

Shower depth:

$$t_{\max} \approx 0.2 \ln(E/\mathrm{GeV}) + 0.7$$

$$L_{95} = t_{\max} + 2.5 \lambda_{\mathrm{att}}$$

with $\lambda_{att} \simeq E[\mathrm{GeV}]^{1.4}$

λ_{att} = exponential attenuation length of the shower tail

Example: 300 GeV pion ...

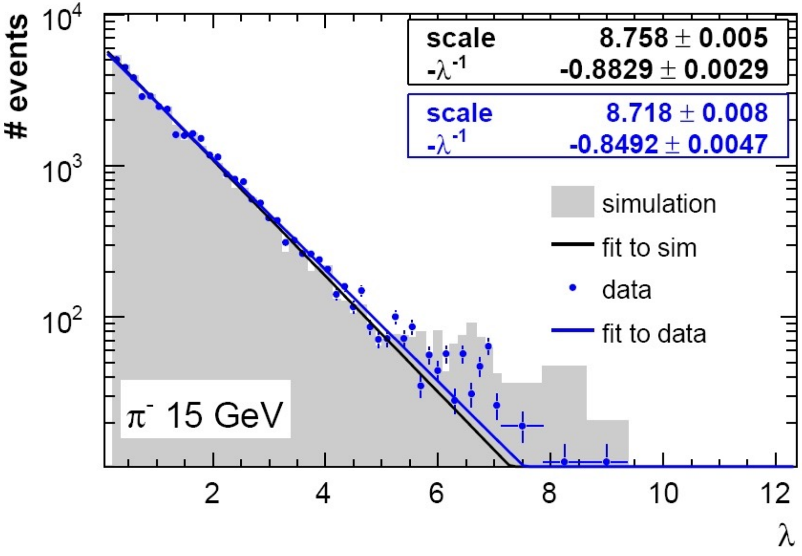
t_{max} = 1.85; L95 = 1.85 + 5.5 ≈ 7.4
[95% within 8λ_{int}; 99% within 11 λ_{int}]

Longitudinal shower profile for 300 GeV π^- interactions in a block of uranium measured from the induced ^{99}Mo radioactivity ...

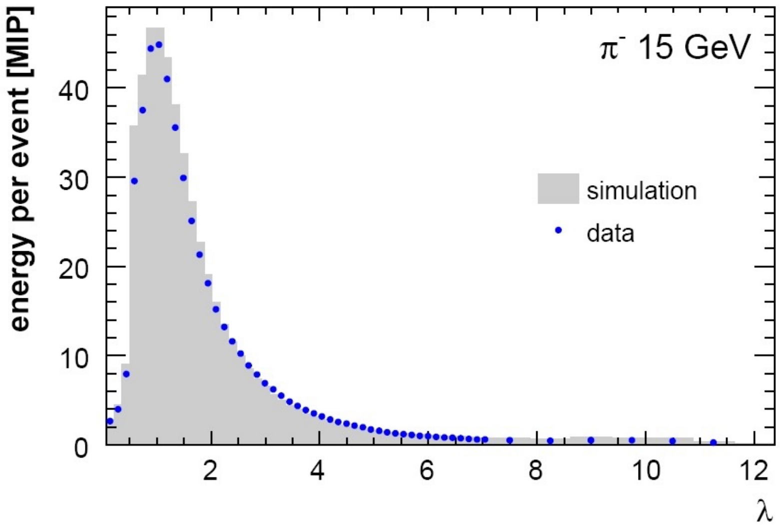
Hadronic shower longitudinal development

- Average longitudinal profile is result of convolution of 2 processes
  - ✓ Depth of first interaction
  - ✓ Longitudinal shower development

Depth of first interaction



Longitudinal shower development

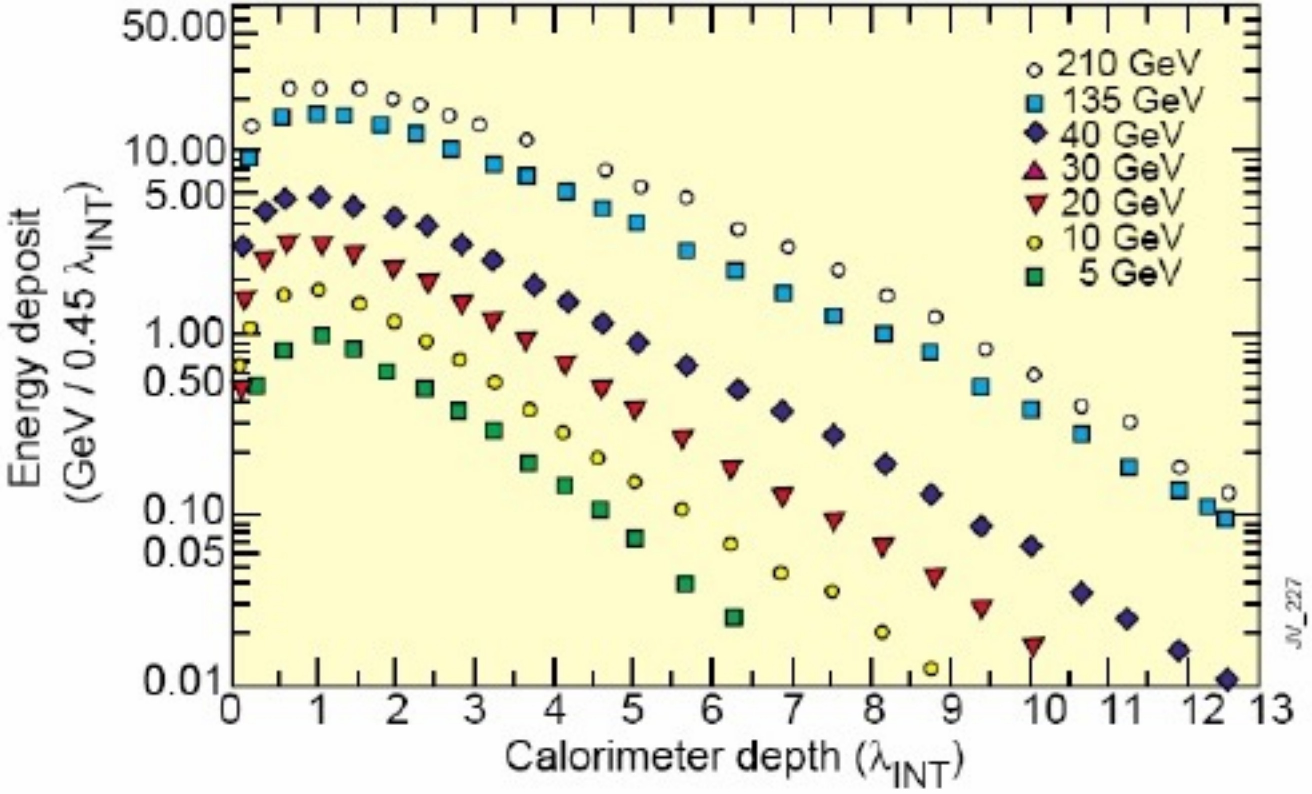


Hadronic shower longitudinal development

Maximum at low depth values due to the EM component in shower that develops more readily due to $X_0 \ll \lambda_{int}$

Hadronic component has long longitudinal development.

Example: for 200 GeV, need > 10 λ_{int} to contain 99% of energy



Exercise: Sizing a hadronic calorimeter

Setup: A 200 GeV pion enters an iron-scintillator HCAL.
Use the material table (slide 9) and the longitudinal profile plots (slide 25).

(a) From the material table, what is λ_{int}(Fe) in cm? [λ_{int} = X g/cm² / ρ g/cm³]

(b) Using the longitudinal formula, estimate the shower maximum depth t_{max} in units of λ_{int} and in cm.

(c) Reading from the energy-deposit plot (slide 25), estimate the total depth in λ_{int} needed to contain 99% of the 200 GeV shower energy. Convert to cm.

(d) A lead ECAL has depth 25 X_0. What is that in cm? Compute the ratio HCAL depth / ECAL depth and comment on the detector design consequence.

Hint: For (b) use the formula from slide 19: $t_{max} = 0.2 \ln(E/\text{GeV}) + 0.7\ [\lambda_{int}]$. For (c) read the 210 GeV curve on slide 25 and estimate when the energy deposit reaches zero.

Exercise: Sizing a hadronic calorimeter

(a) $\lambda_{int}(Fe) = 131.9\ g/cm^2 \div 7.87\ g/cm^3 = 16.8$ cm (both values from slide 9)

(b) $t_{max} = 0.2 \times \ln(200) + 0.7 = 0.2 \times 5.30 + 0.7 \sim 1.76\ \lambda_{int} \sim 1.8\ \lambda_{int}$
→ In cm: 1.8 × 16.8 cm ~ 30 cm (shower peaks only ~30 cm into the calorimeter!)

(c) From slide 25 (210 GeV curve): energy deposit approaches zero at ~11–12 λ_{int}.
→ 99% containment ~ 11–12 λ_{int} ~ 12 × 16.8 cm ~ 200 cm ~ 2 m of iron.

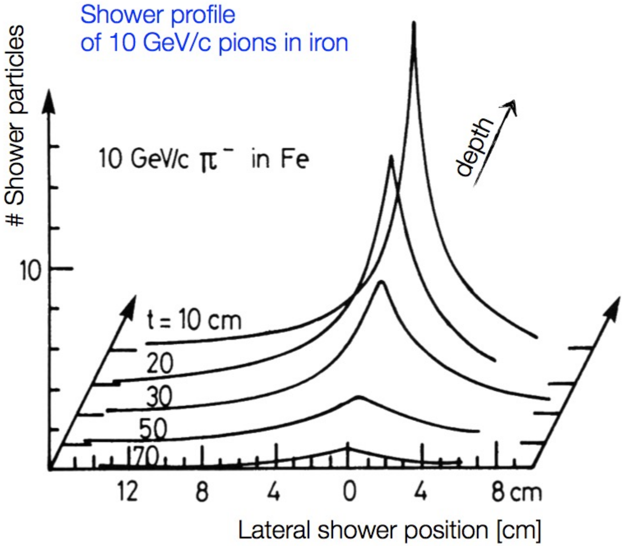
(d) ECAL: $25 \times X_0(Pb) = 25 \times 0.56$ cm = 14 cm. Ratio: 200 cm ÷ 14 cm ~ 14×.
→ For the same fractional containment, an HCAL must be ~14× longer than an ECAL!
→ This is the direct consequence of $\lambda_{int} >> X_0$ for heavy absorbers.

Key takeaway: The large ratio $\lambda_{int}/X_0 \sim 30$ (Pb) forces HCAL to be enormous.
Real LHC HCALs are 7–11 λ_int deep (~1.5–2 m in Fe/Cu).

Hadronic shower lateral development

- Lateral extension set by the (large) transverse momentum of secondary pions (~350 MeV): much broader than EM shower
- Lateral containment
  - ✓ 90% containment radius ~ 1 λ_{int}
  - ✓ 95%: containment radius ~ 2 λ_{int}
    - Compare with EM lateral containment: 95% at ~ 2 R_M ~ 3-5 cm (much smaller!)

- Hadronic shower footprint much larger than EM shower → implications for granularity
  - ✓ Fine granularity less critical for HCAL than for ECAL…
  - ✓ ….BUT particle flow algorithms (Lecture 6) require fine HCAL granularity → tension


Shower profile of 10 GeV/c pions in iron

Lateral profile for 300 GeV π^-

measured from induced radioactivity

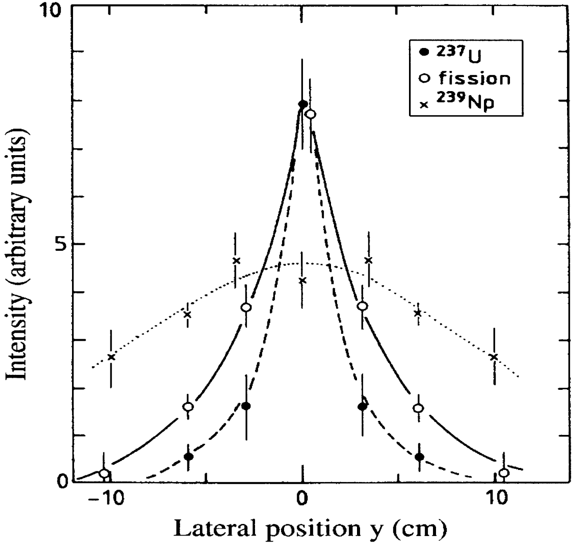
target material ^{238}U

measured at depth 4 λ_{int}

More pions and photons in core

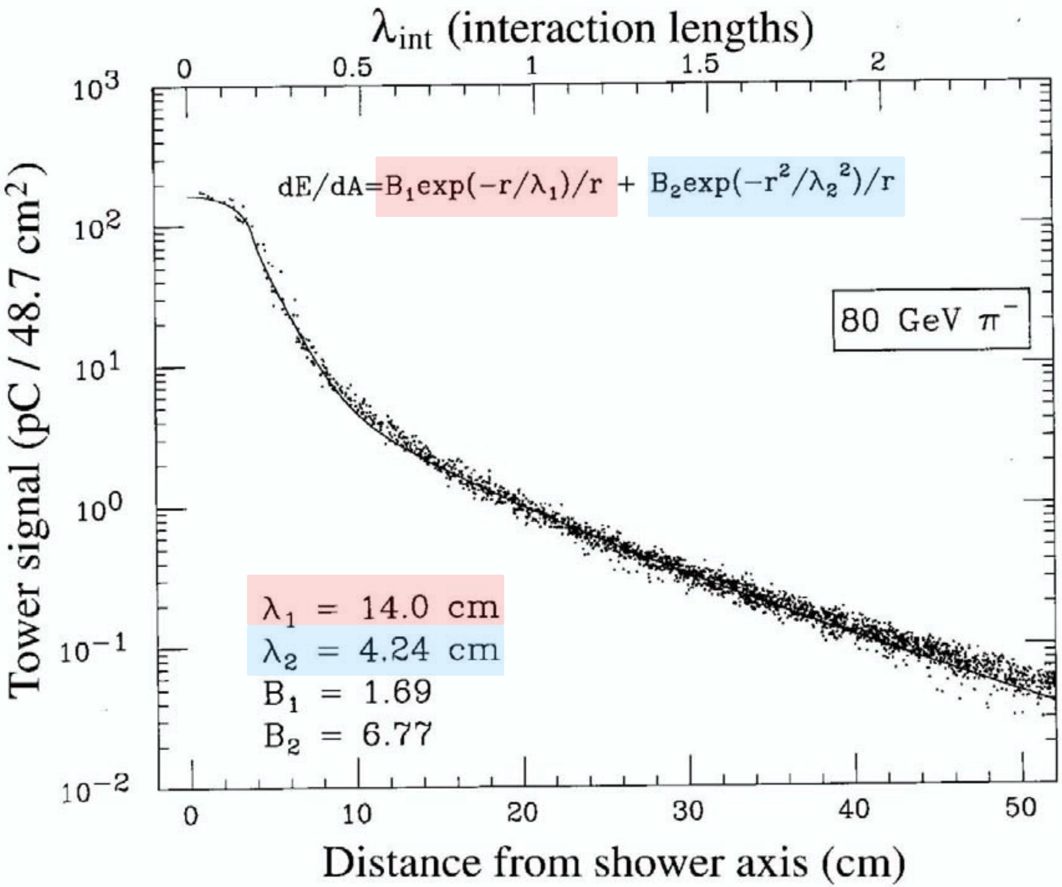
Energetic neutrons and charged pions form a wider core

Thermal neutrons generate broad tail



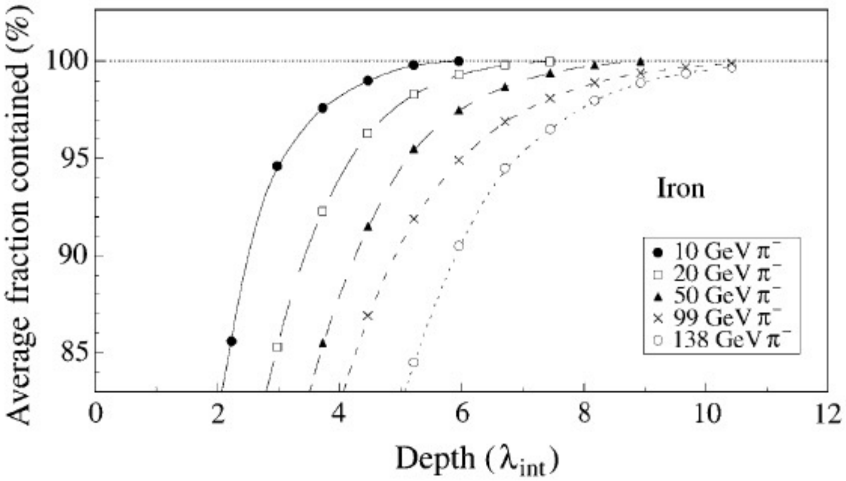
Hadronic shower lateral development

- Lateral shower profile has two components
  - ✓ Electromagnetic core (π^0)
  - ✓ Non-EM halo (mainly non-relativistic shower particles)

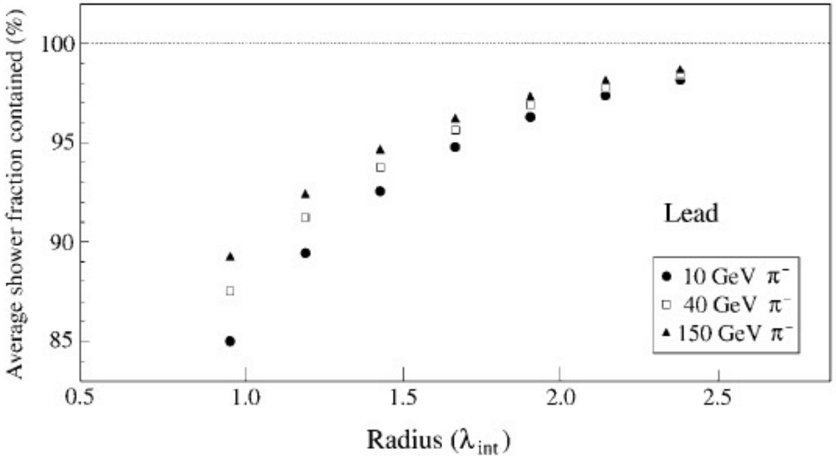


Shower containment

- Depth needed to contain showers increases with ln E
- Lateral leakage decreases as the energy goes up!



- Leakage in principle no problem, can be corrected *on average*…
- … but leakage *fluctuations* are problematic!
  - ✓ Rule of thumb: $\sigma \sim 4\ f_{leak}$
  - ✓ Much smaller for transverse leakage

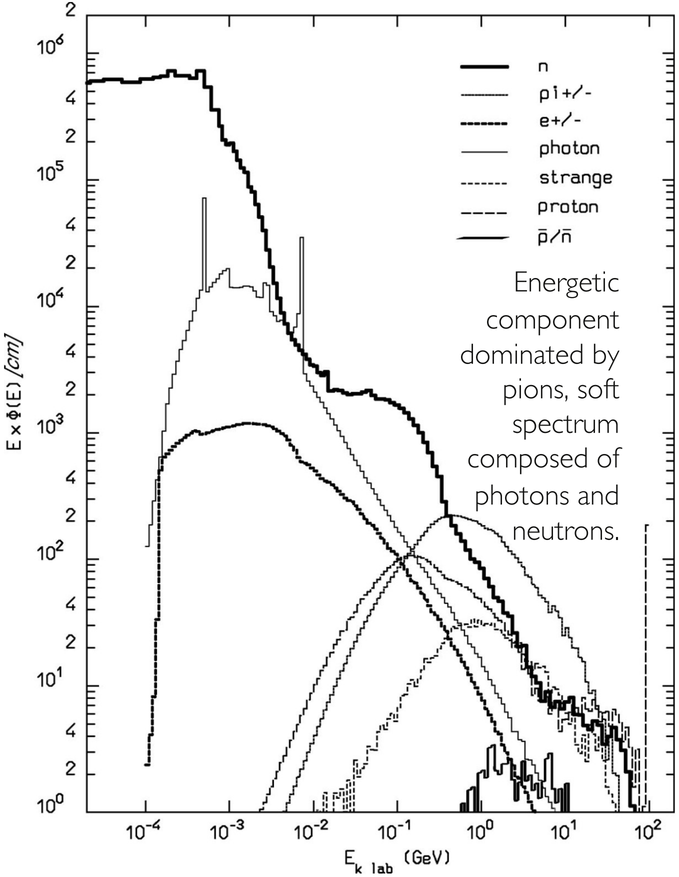


3.3

Hadronic Response & The e/h Ratio

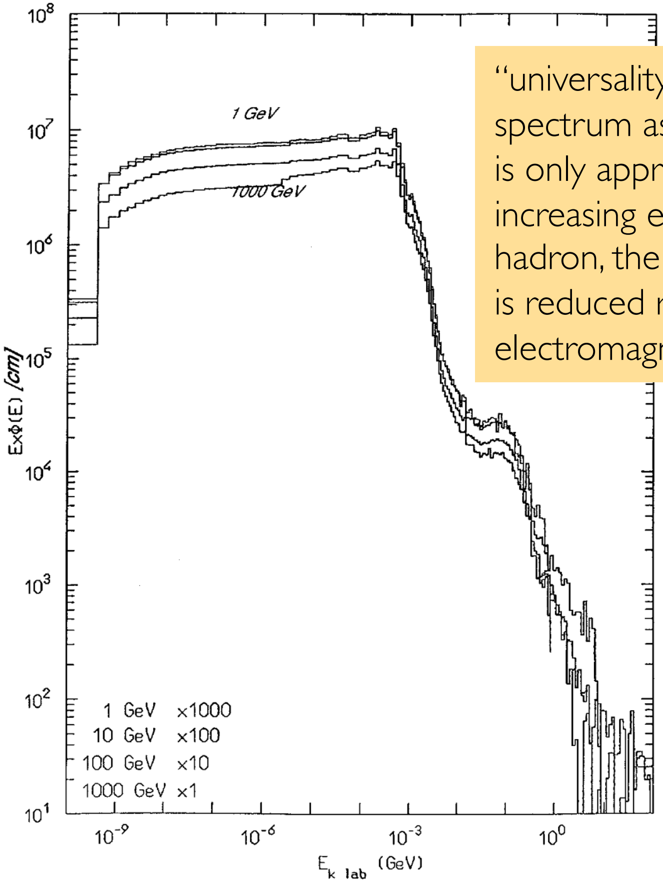
Hadronic shower component spectrum vs E

100 GeV protons in Pb



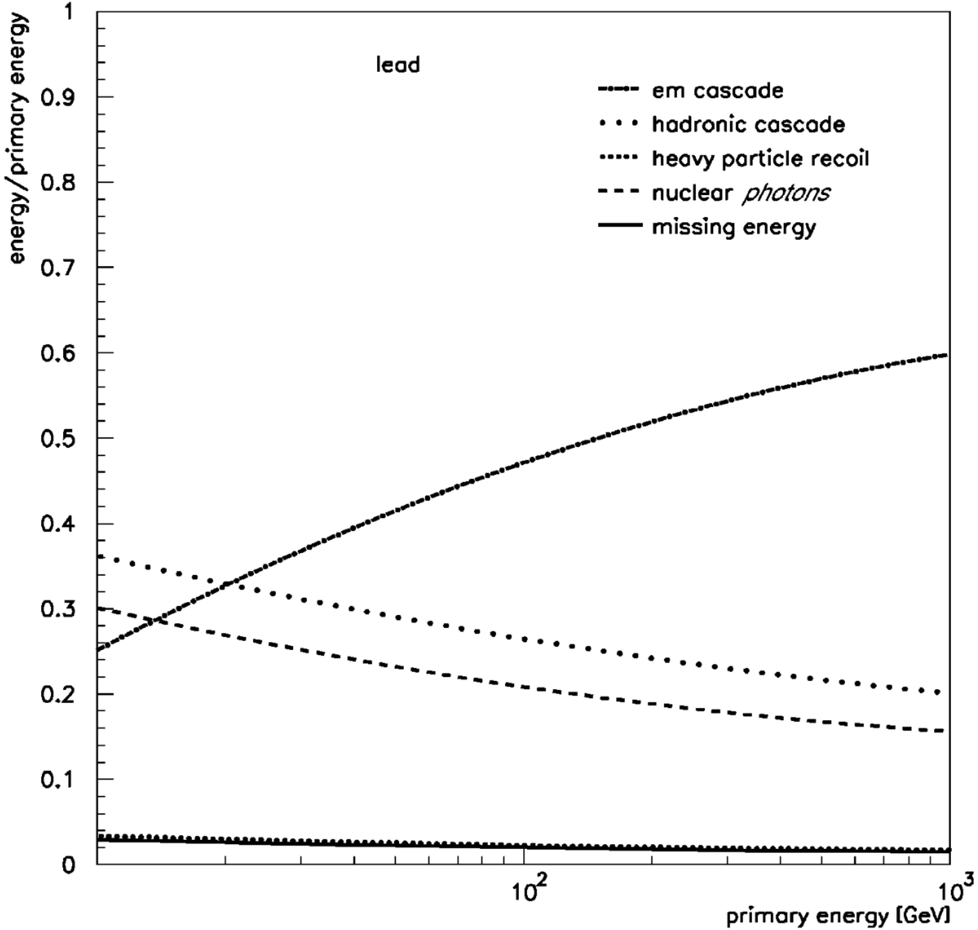
Energetic component dominated by pions, soft spectrum composed of photons and neutrons.

neutrons produced by protons on Pb



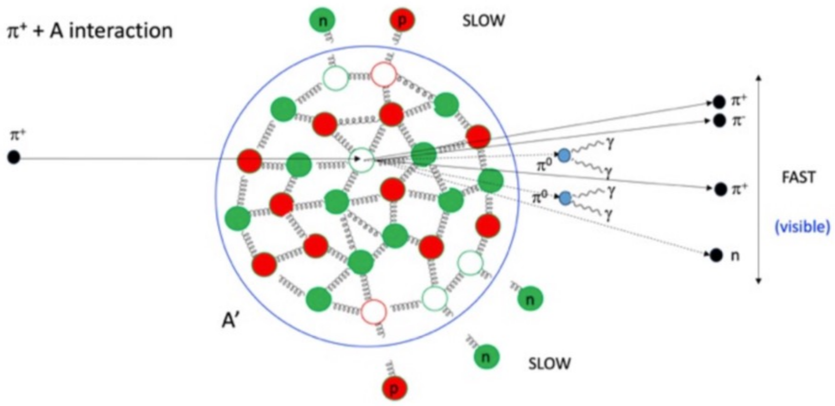
“universality” of shower particle spectrum as a function of energy is only approximate. With increasing energy of the incident hadron, the hadronic component is reduced relative to the electromagnetic component.

Components of proton-initiated cascades in lead

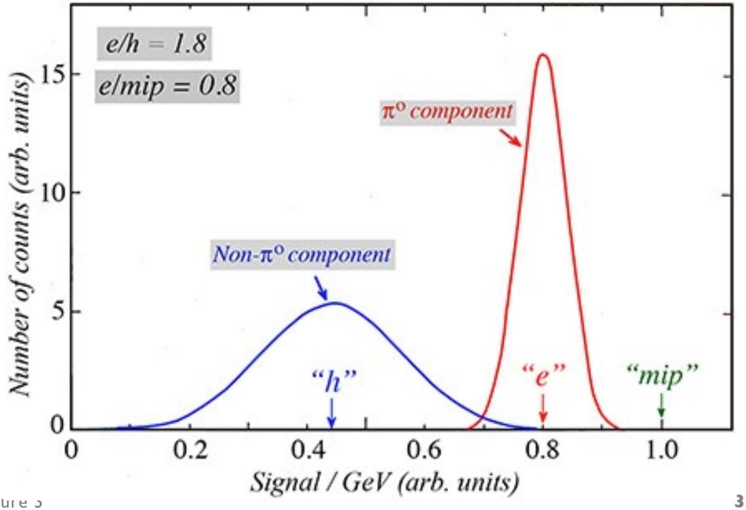


The e/h ratio: fundamental measure of hadronic response

- Hadronic shower has 2 components:
  - ✓ ~1/3 Electromagnetic component
  - ✓ ~2/3 Hadronic component
- Detection efficiency for EM and hadronic component typically different!
  - ✓ This is due to *invisible energy* (e.g. nuclear binding energy losses) affecting hadronic response, and large invisible energy fluctuations



- e/h ratio: intrinsic (energy-independent) measure ratio of response to EM and hadronic particles
  - ✓ e = EM response per unit energy
  - ✓ h = hadronic response per unit energy
  - ✓ e/h > 1
    - EM component is overestimated relative to hadronic ("non-compensation")



From e/h to e/π

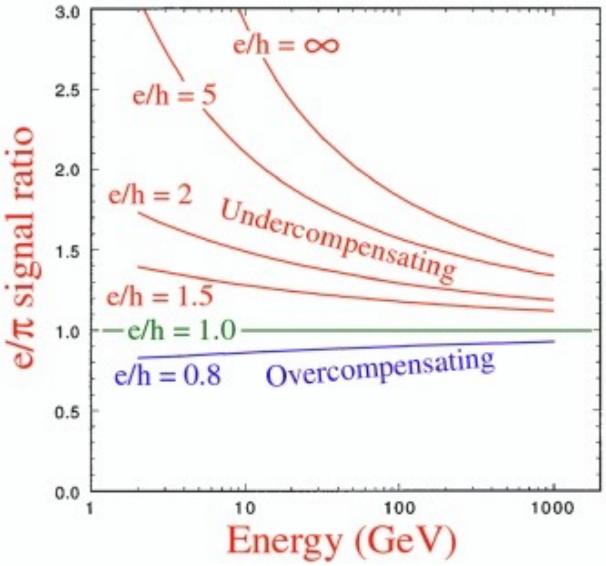
- e/h not directly measurable (can be simulates), gives degree of non-compensation
- e/π: ratio of response between electron-induced and pion-induced shower

$$\frac{e}{\pi} = \frac{e}{f_{\mathrm{em}}e + (1 - f_{\mathrm{em}})h} = \frac{e}{h}\,\frac{1}{1 + f_{\mathrm{em}}\left(\frac{e}{h} - 1\right)}$$

e/h is energy independent
e/π depends on E via $f_{em}(E)$ → non-linearity

Approaches to achieve compensation:

- e/h → 1 (with right choice of materials)
- f_{em} → 1 (high energy limit)



Spoiler: the "compensation" idea looks great on paper for *single hadron* energy reconstruction, but HEP experiments needs to reconstruct *jets*…

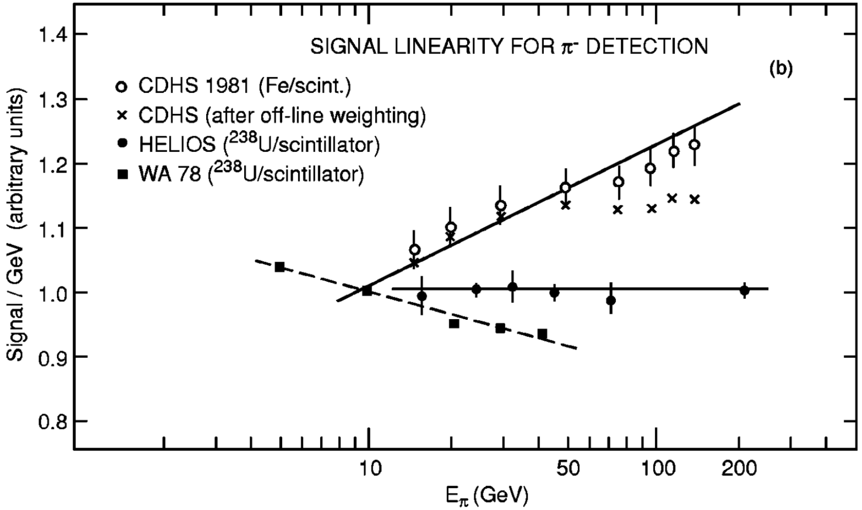
Hadron non-linearity and e/h estimate

- Non-linearity determined by e/h value of the calorimeter
- Measurement of non-linearity is one of the methods to determine e/h
- Assuming linearity for EM showers (e(E) = const):

$$\frac{\pi(E_1)}{\pi(E_2)} = \frac{f_{\mathrm{em}}(E_1) + [1 - f_{\mathrm{em}}(E_1)] \cdot e/h}{f_{\mathrm{em}}(E_2) + [1 - f_{\mathrm{em}}(E_2)] \cdot e/h}$$

For e/h = 1 →

$$\frac{\pi(E_1)}{\pi(E_2)} = 1$$



Other mechanisms relevant to e/h and e/h determination

- Energy deposition mechanisms relevant for the absorption of the non-EM shower energy:
  - ✓ Ionization by charged pions f_{rel} (Relativistic shower component)
  - ✓ spallation protons f_p (non-relativistic shower component).
  - ✓ Kinetic energy carried by evaporation neutrons f_n
- Energy used to release protons and neutrons from calorimeter nuclei, and kinetic energy carried by recoil nuclei do not lead to a calorimeter signal (invisible fraction f_{inv} of non-EM shower energy)
- Total hadron response can be expressed as

$$h = f_{\mathrm{rel}} \cdot \mathrm{rel} + f_p \cdot p + f_n \cdot n + f_{\mathrm{inv}} \cdot \mathrm{inv} \qquad f_{\mathrm{rel}} + f_p + f_n + f_{\mathrm{inv}} = 1$$

Normalizing to mip and ignoring (for now) the invisible component

$$\frac{e}{h} = \frac{e/\mathrm{mip}}{f_{\mathrm{rel}} \cdot \mathrm{rel}/\mathrm{mip} + f_p \cdot p/\mathrm{mip} + f_n \cdot n/\mathrm{mip}}$$

- e/h value can be determined once calorimeter response to the three components of the non-EM shower are known

Other mechanisms relevant to e/h and e/h determination

- e/h value can be determined once calorimeter response to the three components of the non-EM shower are known

$$\frac{e}{h} = \frac{e/\mathrm{mip}}{f_{\mathrm{rel}} \cdot \mathrm{rel}/\mathrm{mip} + f_p \cdot p/\mathrm{mip} + f_n \cdot n/\mathrm{mip}}$$

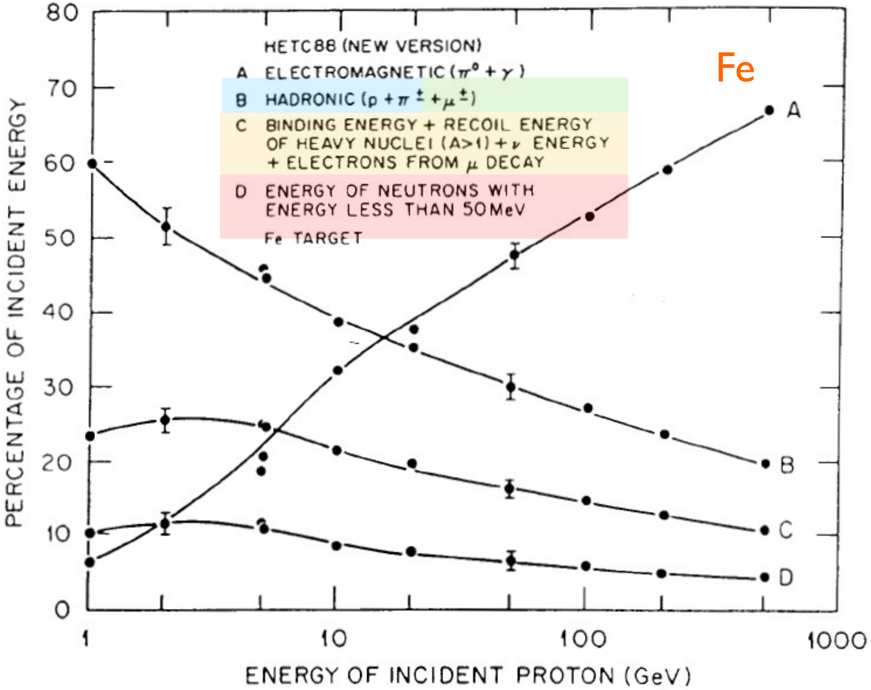
✓ Relativistic charged hadrons
  - Even if relativistic, these particles resemble mip in their ionization losses (rel/mip = 1)

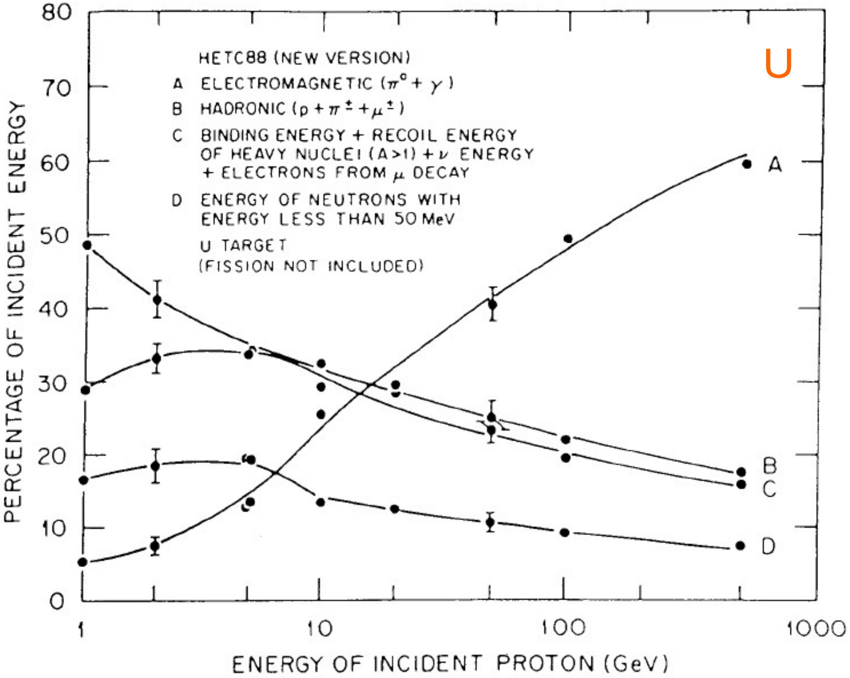
✓ Spallation protons
  - Can compensate by more efficient sampling (p/mip>1)
  - Should account for signal saturation

✓ Evaporation neutrons
  - (n → n'γ) inelastic scattering: not very important
  - (n → n') elastic scattering: most interesting capture
  - (n → γ) capture (thermal): lots of energy, but process is slow (μs)

Energy fraction in hadron shower

$$h = f_{\mathrm{rel}} \cdot \mathrm{rel} + f_p \cdot p + f_n \cdot n + f_{\mathrm{inv}} \cdot \mathrm{inv}$$





3.4

Shower Time Evolution and Signal Shaping

Time structure of hadronic showers

- Hadronic shower signal has a complex time structure, very different from EM!
  - ✓ Fast component: EM sub-showers (from π^0), signal in < 10 ns
  - ✓ Intermediate: prompt pion/proton component, 10-200 ns
  - ✓ Slow component: neutron capture, de-excitation photons, up to microseconds

Practical implication: electronics shaping time determines how much signal is collected

- *Short shaping (LHC: 25-50 ns): accept some signal loss for speed*
- *→ Lecture 7: electronics shaping and optimal filtering*

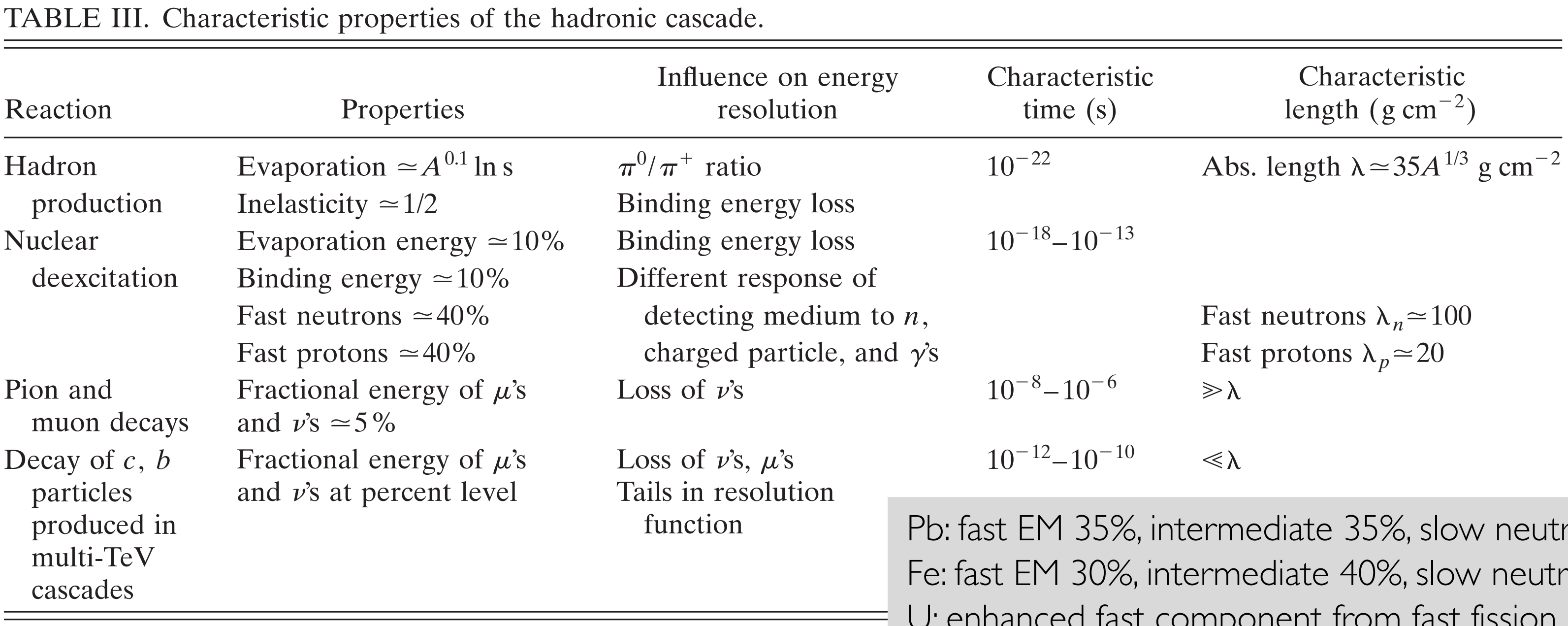
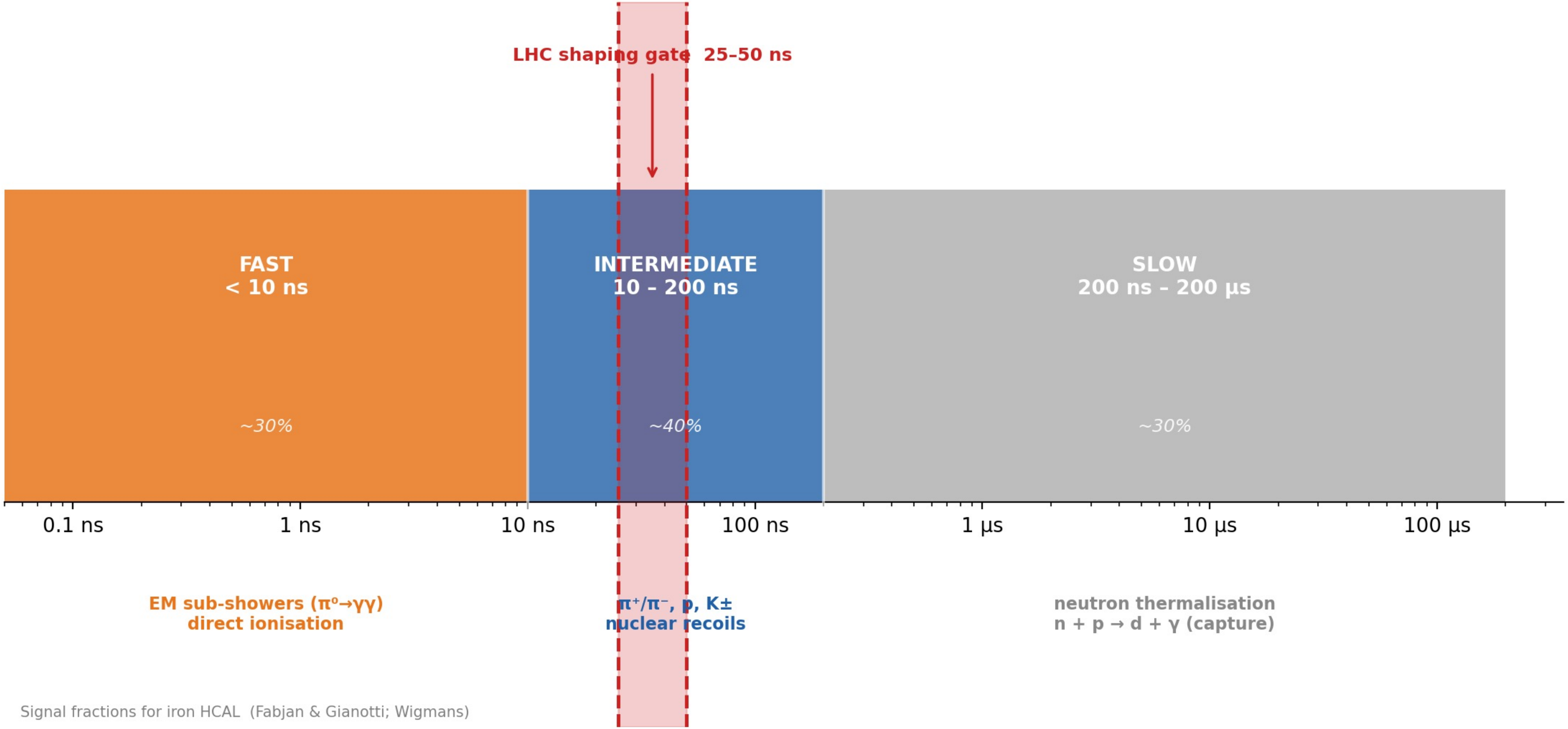


TABLE III. Characteristic properties of the hadronic cascade.

Reaction	Properties	Influence on energy resolution	Characteristic time (s)	Characteristic length (g cm^{-2})
Hadron production	Evaporation $\simeq A^{0.1} \ln s$ Inelasticity $\simeq 1/2$	π^0/π^+ ratio Binding energy loss	10^{-22}	Abs. length $\lambda \simeq 35 A^{1/3}$ g cm^{-2}
Nuclear deexcitation	Evaporation energy $\simeq 10\%$ Binding energy $\simeq 10\%$ Fast neutrons $\simeq 40\%$ Fast protons $\simeq 40\%$	Binding energy loss Different response of detecting medium to n, charged particle, and γ's	10^{-18}–10^{-13}	Fast neutrons $\lambda_n \simeq 100$ Fast protons $\lambda_p \simeq 20$
Pion and muon decays	Fractional energy of μ's and ν's $\simeq 5\%$	Loss of ν's	10^{-8}–10^{-6}	$\gg \lambda$
Decay of c, b particles produced in multi-TeV cascades	Fractional energy of μ's and ν's at percent level	Loss of ν's, μ's Tails in resolution function	10^{-12}–10^{-10}	$\ll \lambda$

Pb: fast EM 35%, intermediate 35%, slow neutrons 30%
Fe: fast EM 30%, intermediate 40%, slow neutrons 30%
U: enhanced fast component from fast fission neutrons

Time structure of hadronic showers



LHC shaping gate 25–50 ns

FAST
< 10 ns
~30%

INTERMEDIATE
10 – 200 ns
~40%

SLOW
200 ns – 200 µs
~30%

EM sub-showers (π⁰→γγ)
direct ionisation

π⁺/π⁻, p, K±
nuclear recoils

neutron thermalisation
n + p → d + γ (capture)

Signal fractions for iron HCAL (Fabjan & Gianotti; Wigmans)

3.5

Bridge: What Would a Perfect HCAL Need?

Recap: physics lessons important for calorimetry

- In absorption process, most of the energy is deposited by very soft shower particles
  - ✓ Electromagnetic showers
    - ~3/4 of the energy deposited by electrons ($e^{\pm}$): pair-production secondaries and Compton photoelectrons (~half of the electron component)
    - ~1/4 by photons below the pair-production threshold (photoelectric absorption)
      - These are isotropic, have forgotten direction of incoming particle
    - The typical shower particle is a 1 MeV electron, range < 1 mm
      - Important consequences for sampling calorimetry
  - ✓ Hadron showers
    - Typical shower particles are a 50 - 100 MeV proton and a 3 MeV evaporation neutron
    - Range of 100 MeV proton is 1 - 2 cm
    - Neutrons travel typically several cm
    - What they do depends crucially on details of the absorber

Recap: hadronic physics facts for calorimeter designers

- The three fundamental hadronic calorimetry challenges:
  - ✓ 1. Shower scale: $\lambda_{int} >> X_0$
    - HCAL must be large (10-12 λ_{int} ~ 1.5-2 m in Fe)
  - ✓ 2. Invisible energy: ~40% of hadronic energy is undetectable
    - fundamental response loss
  - ✓ 3. Fluctuating EM fraction: f_{em} varies event-by-event
    - non-Gaussian resolution
- And one additional complication
  - ✓ 4. Time structure: slow neutron component
    - signal shaping must integrate carefully

- Question for Lecture 4: given these four facts, what would a perfect HCAL look like?

Bridge question: designing the ideal HCAL

- What properties should the ideal hadron calorimeter have?
  - ✓ A. Depth sufficient to contain the shower longitudinally
  - ✓ B. A mechanism to treat the EM and hadronic components equally → e/h = 1
  - ✓ C. Good sampling of the shower over many layers
  - ✓ D. Fast enough readout to capture the full signal within the bunch crossing time

- Lecture 4 will quantify each of these (and derive resolution properties)
- Answer to B is *compensation:* a clever piece of calorimeter physics
  - ✓ We saw some aspects of the issue in this lecture, we'll tackle some possible solutions in next one

Capstone exercise: Why is hadronic calorimetry hard?

Setup: Using the physics covered in today's lecture, answer the following questions about the fundamental limitations of hadronic calorimeters.

(a) Using the f_{em} power-law formula from slide 17 (E_0 = 1 GeV, k = 0.85), estimate f_{em} for a 100 GeV pion shower.

(b) EM sub-showers develop on the scale of $X_0 \sim 0.56$ cm (Pb). HCAL cells are of order $\lambda_{int} \sim 17$ cm. Does the HCAL measure the EM and hadronic components separately? What is the consequence for energy reconstruction?

(c) From the timing table (slide 41), assume the LHC shaping gate integrates ~50 ns. Which time components are captured? Which are largely missed? Estimate the fraction of signal lost.

(d) Synthesise (a)–(c): list two independent reasons why a non-compensating calorimeter (e/h != 1) has poor energy resolution for hadrons.

Capstone exercise: Solutions (a, b)

(a) $f_{em} = 1 - (100/1)^{(0.85-1)} = 1 - 100^{(-0.15)} = 1 - 10^{(-0.30)} = 1 - 0.50 \sim 50\%$.
→ At 100 GeV, about half the shower energy is in the EM component. This fraction varies event-by-event (σ_f ~ 10–15%), which is the dominant contribution to hadronic energy resolution.

(b) No: $X_0 = 0.56$ cm << λ_int = 17 cm. The HCAL cell is ~30× coarser than X_0.
Consequence: EM sub-showers and hadronic deposits are summed in the same readout cell. The detector cannot measure f_em event-by-event. Since the EM response (e) and the hadronic response (h) are different in a non-compensating calorimeter, the total signal $S/e = f_{em} + (h/e)(1 - f_{em})$ fluctuates with f_em. This is irreducible.

Response formula: $S/e = f_{em} + (h/e)(1 - f_{em})$. With $e/h = 1.4$ (non-compensating Fe/scint) and $f_{em} = 0.50$: $S/e = 0.50 + (1/1.4)\times0.50 = 0.50 + 0.36 = 0.86$. The calorimeter undershoots by 14% on average, and the undershoot fluctuates with f_{em}.

Capstone exercise: Solutions (c, d)

(c) From slide 41-42: fast EM component (<10 ns, ~30%) and intermediate hadronic component (10–200 ns, ~40%) are captured within 50 ns → ~70% of signal captured.
Slow thermal-neutron component (200 ns–200 μs, ~30%) is largely missed. Signal loss ~30%, with large event-by-event fluctuations in the slow fraction.

(d) Two independent reasons for poor hadronic energy resolution in a non-compensating calorimeter:

1. f_{em} fluctuations (from a): f_{em} varies event-by-event (~10–15%). Since e != h, the total signal $S/e = f_{em} + (h/e)(1-f_{em})$ fluctuates even at fixed incident energy.

2. Invisible energy + slow-neutron loss (from b, c): nuclear binding energy (~30–40% of hadronic component) is permanently invisible. The slow-neutron fraction captured by the gate also fluctuates event-by-event, adding non-Gaussian tails to the resolution.

→ Both effects are reduced (in principle) by compensation (e/h = 1): equal response to EM and hadronic deposits removes the f_em-driven fluctuation.

What did we learn today?

- **Week 2 (Physics depth)**
  - ✓ Lecture 3: Hadronic shower physics
    - *3.1 Nuclear Interactions and Interaction Length*
      - *$\lambda_{int} \gg X_0$ forces HCAL to be much larger than ECAL; ~40% of hadronic energy in heavy absorbers is invisible (nuclear binding energy)*
    - *3.2 Hadronic Shower Structure & Development*
      - *Two-component shower (EM + hadronic); f_{em} grows with E and fluctuates event-by-event; longitudinal containment requires 10–12 λ_{int}; lateral halo extends to ~2 λ_{int}*
    - *3.3 Hadronic Response & the e/h ratio*
      - *e/h != 1 (non-compensation): hadronic response is non-linear and the fluctuations in f_{em} dominate hadronic energy resolution*
    - *3.4 Shower Time Evolution and Signal Shaping*
      - *Fast EM (<10 ns), intermediate π/p (10–200 ns), slow neutrons (up to μs); LHC 25–50 ns gate captures ~70%, losing ~30% of signal*
    - *3.5 Bridge: What Would a Perfect HCAL Need?*
      - *Four challenges → compensation (e/h = 1), deep containment, fine granularity, fast readout: exactly the agenda of Lecture 4…*
  - ✓ Lecture 4: Energy resolution from first principles