Calorimetry

in particle physics experiments

2. Electromagnetic Interactions and Shower Physics

Course roadmap

- **Week 1 (Foundations)**
  - ✓ Lecture 1: Why calorimetry?
  - ✓ Lecture 2: EM shower physics
- **Week 2 (Physics depth)**
  - ✓ Lecture 3: Hadronic shower physics
  - ✓ Lecture 4: Energy resolution from first principles
- **Week 3 (Technology)**
  - ✓ Lecture 5: Calorimeter Technologies (real-life EM and Hadronic calorimeters)
  - ✓ Lecture 6: Calorimeter Design
- **Week 4 (Systems & Future)**
  - ✓ Lecture 7: Signal chain, readout, calibration
  - ✓ Lecture 8: Future calorimetry

What particle do we measure with calorimeters?

Note that at energy > 1 TeV muon calorimetry becomes possible as muons in lead/iron undergo interaction processes where the energy loss is proportional to the muon energy

Today's Lecture

- **Week 1 (Foundations)**
  - ✓ Lecture 1: Why calorimetry?
  - ✓ Lecture 2: EM shower physics
    - *2.1 Photon Interactions in Matter*
    - *2.2 Charged-Particle Interactions in Matter*
      - *"Heavy" particle ($M \gg m_e$; e.g. muons)*
      - *Electrons*
    - *2.3 Electromagnetic Shower development*
    - *2.4 From Particle Physics To Calorimeter Design*

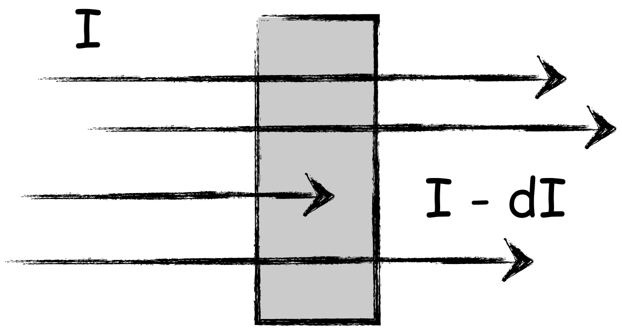
2.1

Photon Interactions in Matter

Characteristics of photon interactions with matter

- A single interaction remove photon from beam!

- Possible interactions:
  - ✓ Photoelectric effect
  - ✓ Compton scattering
  - ✓ Pair production



- ✓ Rayleigh Scattering ($\gamma A \to \gamma A$; A = atom; coherent)
- ✓ Thomson Scattering ($\gamma e \to \gamma e$; elastic scattering)
- ✓ Photo Nuclear Absorption ($\gamma K \to pK/nK$)
- ✓ Nuclear Resonance Scattering ($\gamma K \to K^* \to \gamma K$)
- ✓ Delbruck Scattering ($\gamma K \to \gamma K$)
- ✓ Hadron Pair production ($\gamma K \to h^+h^- K$)

x = mass thickness [g/cm^2]
μ = mass attenuation coefficient [cm^2/g] (depends on E, Z, ρ)

$$dI = -\mu I dx$$

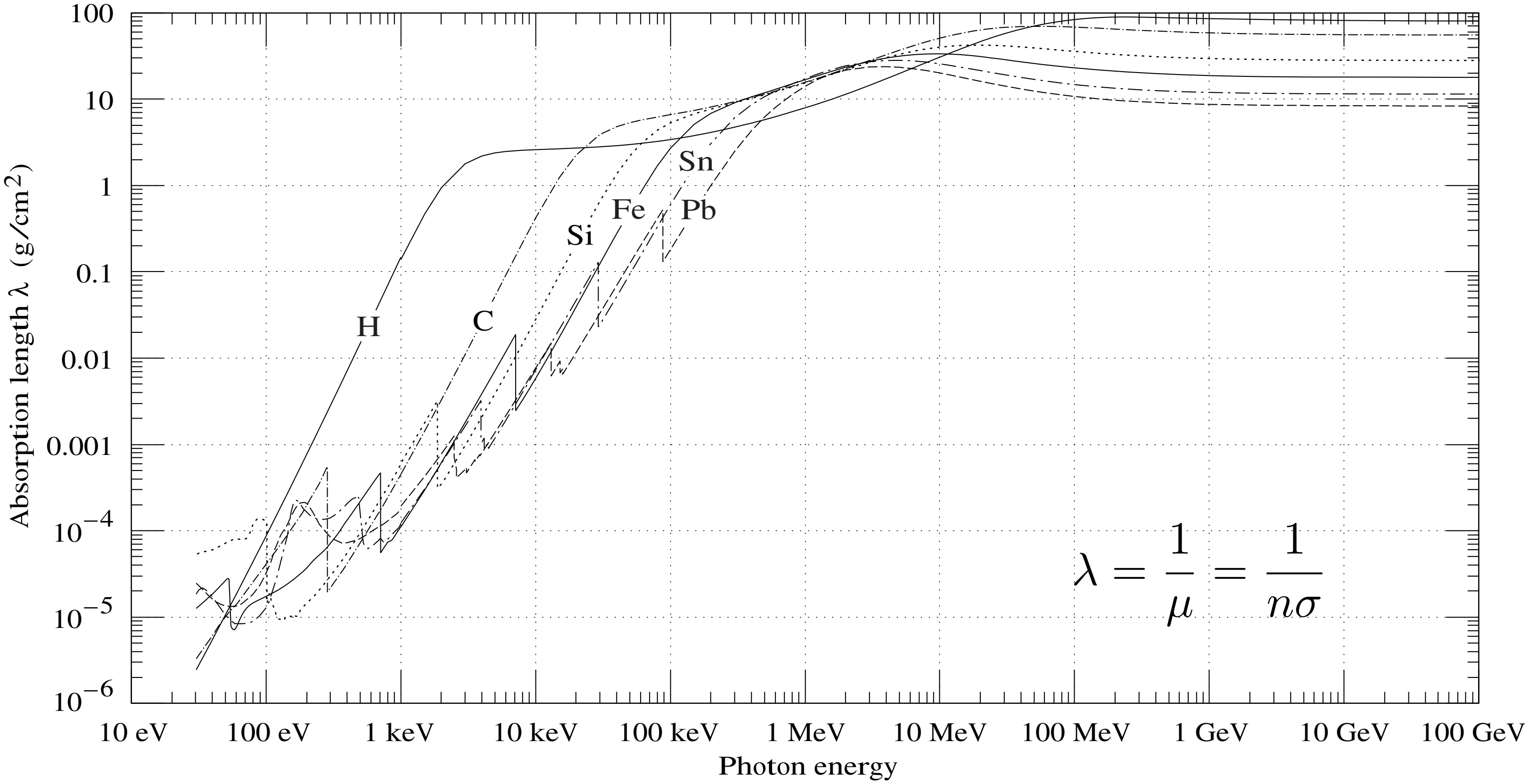
Beer-Lamber law

$$I(x) = I_0 e^{-\mu x}$$

mean free path [g/cm^2]

$$\lambda = \frac{1}{\mu} = \frac{1}{n\sigma}$$

Absorption length vs Energy



$$\lambda = \frac{1}{\mu} = \frac{1}{n\sigma}$$

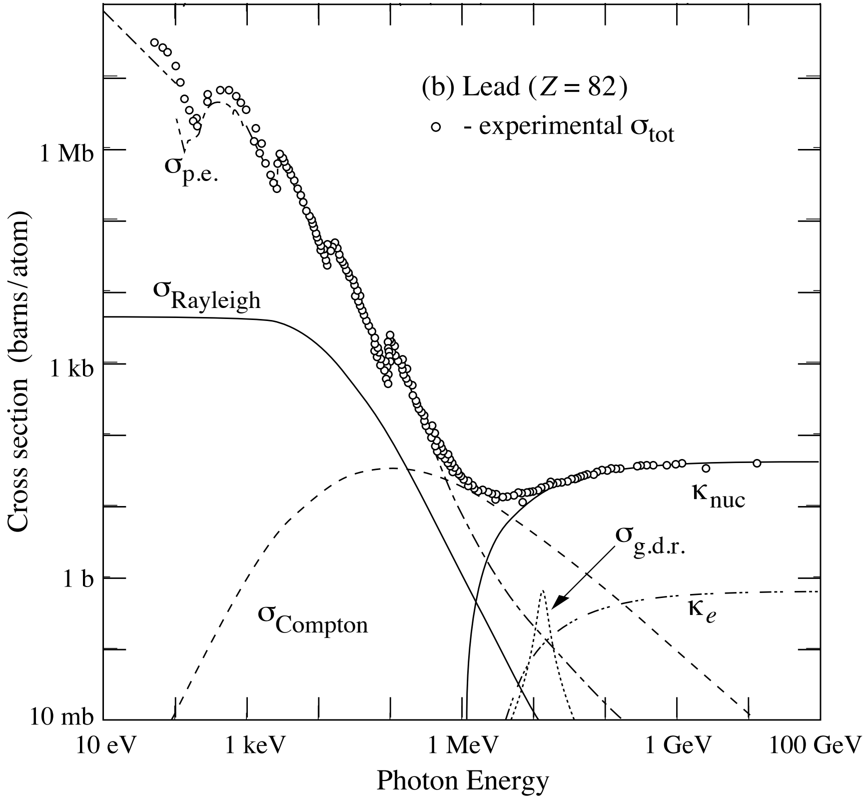
Absorption length & interaction cross section

- Three dominant processes, cross-section depends on E and traversed material Z
  - ✓ Photoelectric effect
    - $\sigma \sim Z^5 / E^{3.5}$
    - dominant at low E (<100 keV)
  - ✓ Compton scattering
    - $\sigma \sim Z / E$
    - dominant at intermediate E (0.1–10 MeV)
  - ✓ Pair production
    - $\sigma \sim Z^2 \ln(E)$
    - dominant at high E (>10 MeV in heavy materials)

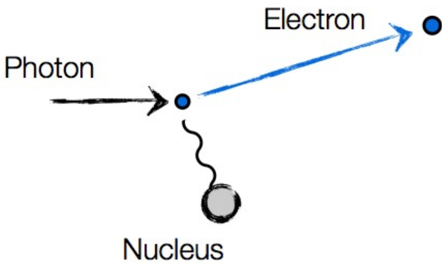
$$I(x) = I_0 e^{-\mu x}$$

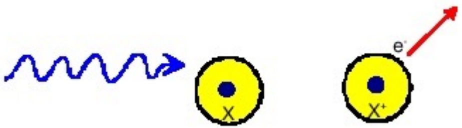
$$\mu = n\sigma = \frac{N_A}{A}\sigma_x$$

N_A = Avogadro's number
A = atomic weight
σ = total cross-section for photon absorption



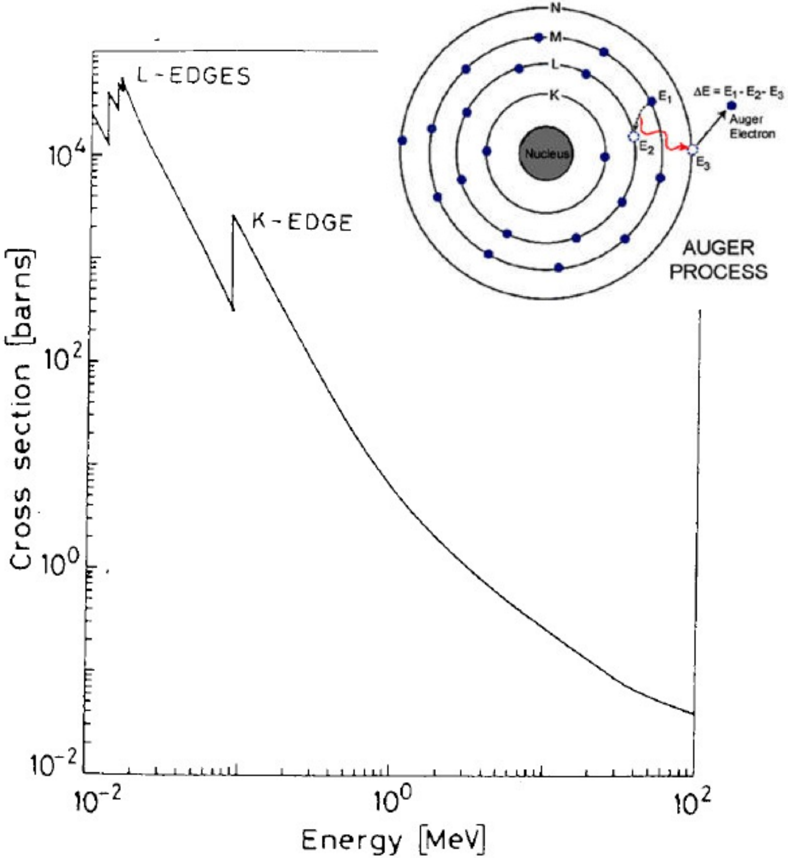
Photoelectric effect





$$\gamma + \text{atom} \rightarrow \text{atom}^{+} + e^{-}$$

- Can be considered as an interaction between a photon and an atom *as a whole*
- Happens if a photon has energy $E_\gamma > E_b$ (binding energy of an electron)
  - ✓ Photon energy is fully transferred to the electron
  - ✓ Electron is ejected with energy $T = E_\gamma - E_b$
- Discontinuities in the cross-section due to discrete energies E_b of atomic electrons
  - ✓ modulations at $E\gamma = Eb = E_L, E_K, \ldots$
- Dominating process at low γ energies (<100 keV)
  - ✓ Especially in high Z material



Following emission of "photoelectron", atom is in an excited state. De-excitation occurs via two effects:

- Fluorescence: $\text{Atom}^{*+} \rightarrow \text{Atom}^{*+} + \gamma$ (X-rays)
- Auger effect: $\text{Atom}^{*+} \rightarrow \text{Atom}^{*++} + e^-$ (Auger electrons, E ~ 10 KeV)

Photoelectric effect

- "Low" energy ($I_0 < E_\gamma << m_e$)

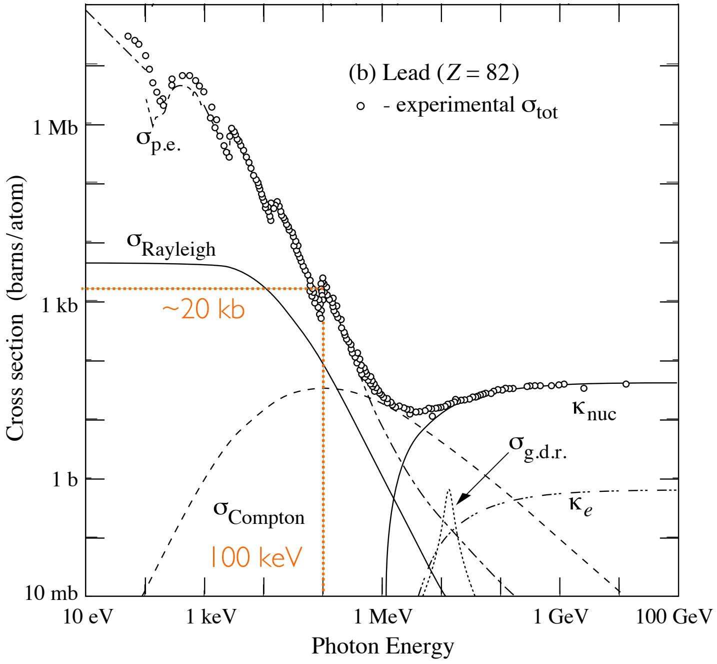
$$\sigma_{\rm ph} = \alpha \pi a_B Z^5 (I_0/E_\gamma)^{7/2} \propto Z^5 E_\gamma^{-7/2}$$

- "High" energy ($E_\gamma >> m_e$)

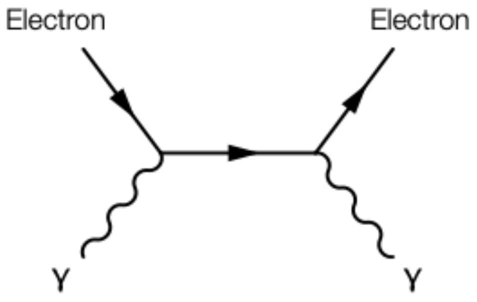
$$\sigma_{\rm ph} = 2\pi r_e^2 \alpha^4 Z^5 (m_e/E_\gamma) \propto Z^5 E_\gamma^{-1}$$

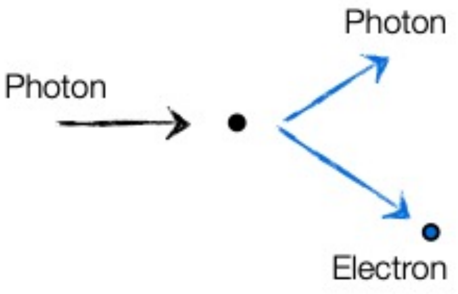
- Example
  - ✓ a_B = 0.53 10^{-10} m
  - ✓ I_0 = 88 keV (Pb K-edge)
  - ✓ Z = 82 (Pb)
  - ✓ E_γ = 100 KeV (just above the K-edge)
  - ✓ σ_{ph}(Pb) ~ 20 kb (PDG)

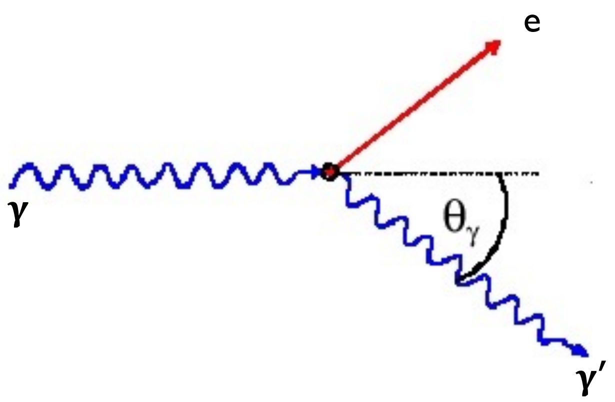
I_0 = ionization potential
α = fine structure constant
$r_e = \alpha/m_e$ = classical electron radius
$a_B = 1/(m_e\alpha)$ = Bohr radius
$r_e = \alpha^2 a_B$



Compton scattering







$$\gamma + e \rightarrow \gamma' + e$$

$$\frac{E_{\gamma'}}{E_\gamma} = \frac{1}{1 + (E_\gamma / m_e)\cos\theta_\gamma}$$

$$E_k^e = E_\gamma - E_{\gamma'} = E_\gamma \frac{(1-\cos\theta_\gamma)(E_\gamma/m_e)}{1 + (E_\gamma/m_e)(1-\cos\theta_\gamma)}$$

- Electrons in matter are bound, but when E_γ >> binding electron energy electron can be considered as "free"
- Compton scattering = scattering of γ on quasi-free atomic electrons
- A fraction of E_γ is transferred to the electron, $E_{\gamma'} < E_\gamma$
- Dominating process at intermediate γ energies (0.1–10 MeV)

- $\theta_\gamma = 0$ → forward scattering → $E_k^e = 0$
  - ✓ Initial photon can give all its energy to final photon…
- $\theta_\gamma = \pi$ → backward scattering → $E_k^e = E_{max}$; $E_{\gamma'} = E_{min}$
  - ✓ … but not to electron!
  - ✓ Photon cannot be completely absorbed by Compton Scattering

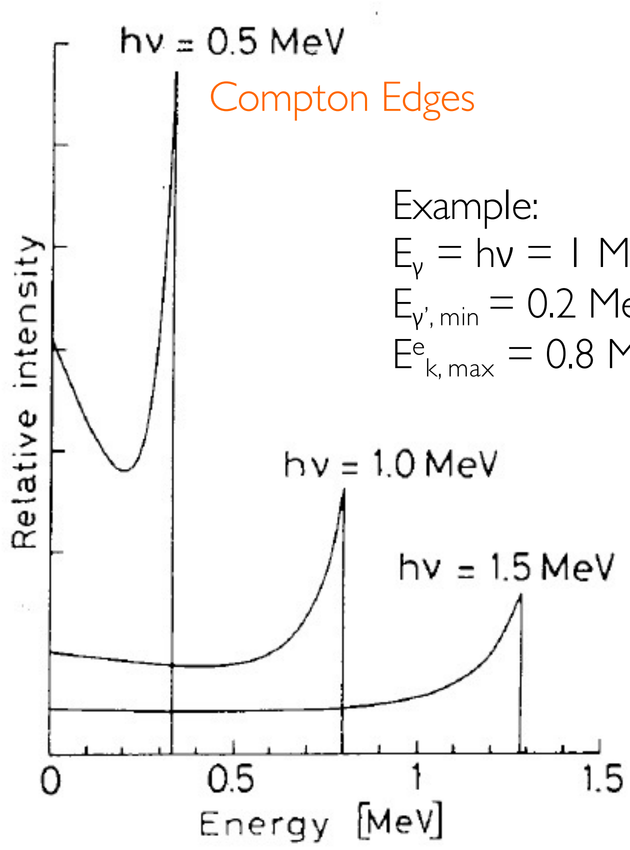
Compton scattering

- $\theta_\gamma = \pi \rightarrow$ backward scattering

$$E_{\gamma',\min} = E_\gamma \frac{1}{1+2(E_\gamma/m_e)}$$

$$E^e_{k,\max} = E_\gamma \frac{2(E_\gamma/m_e)}{1+2(E_\gamma/m_e)}$$

- Complete transfer of γ energy to electron via Compton scattering is not possible
  - ✓ Important for single photon detection: photon cannot be completely absorbed and the scattered electron misses a small amount of initial energy



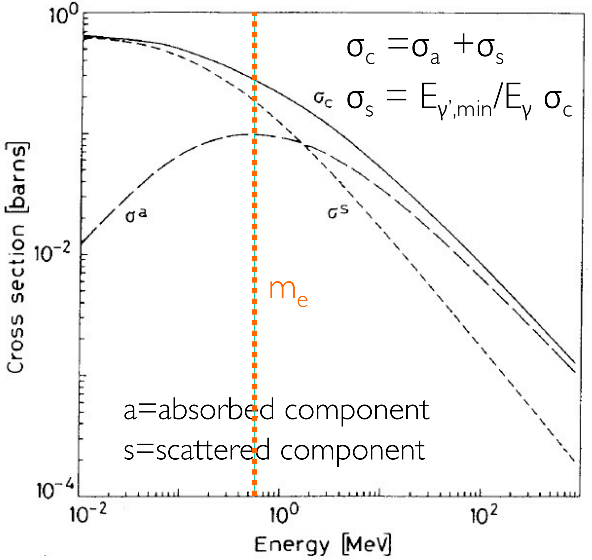
Example:
E_γ = hν = 1 MeV
$E_{\gamma', \min}$ = 0.2 MeV
$E^e_{k,\max}$ = 0.8 MeV

Compton scattering

- Klein-Nishina Formula (LO QED) for differential and total cross sections

$$\frac{d\sigma_c^e}{d\Omega} = \frac{r_e^2}{2}\frac{1+\cos^2\theta_\gamma}{(1+\epsilon(1-\cos\theta_\gamma))^2}\left(1+\frac{\epsilon^2(1-\cos\theta_\gamma)^2}{(1+\cos^2\theta_\gamma)(1+\epsilon(1-\cos\theta_\gamma))}\right) \quad (\textit{per electron}) \qquad \epsilon = \frac{E_\gamma}{m_e}$$

$$\sigma_c^e = 2\pi r_e^2\left(\left(\frac{1+\epsilon}{\epsilon^2}\right)\left\{\frac{2(1+\epsilon)}{1+2\epsilon} - \frac{1}{\epsilon}\ln(1+2\epsilon)\right\} + \frac{1}{2\epsilon}\ln(1+2\epsilon) - \frac{1+3\epsilon}{(1+2\epsilon)^2}\right) \quad (\textit{per electron})$$



- "Low" energy ($E_\gamma << m_e$)

$$\sigma_C = \sigma_T(1 - E_\gamma/m_e)$$

Thompson cross-section

$$\sigma_T = \frac{8\pi}{3r_e^2} = 0.66 \text{ barn}$$

- "High" energy ($E_\gamma >> m_e$)

$$\sigma_C \propto \frac{\log E_\gamma}{E_\gamma}$$

$$\sigma_C^{atom} = Z\sigma_C^e$$

Pair production

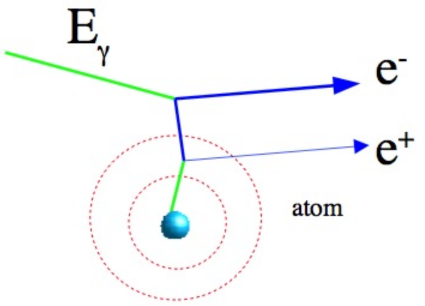
- An electron-positron pair can be created when (and only when) a photon passes by the Coulomb field of a nucleus or atomic electron
  - ✓ This is needed for momentum conservation!
  - ✓ It is a threshold process
    - Threshold: e^+e^- rest masses + recoil energy of nucleus or electron

$$E_\gamma > 2m_e \left(1 + \frac{m_e}{m_X}\right)$$

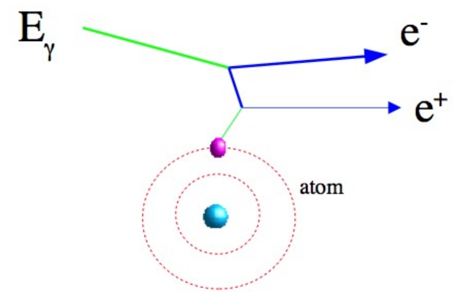
$m_X = m_N$ or m_e

$E_\gamma \sim 2\, m_e$ near nucleus
$E_\gamma = 4\, m_e$ near electron

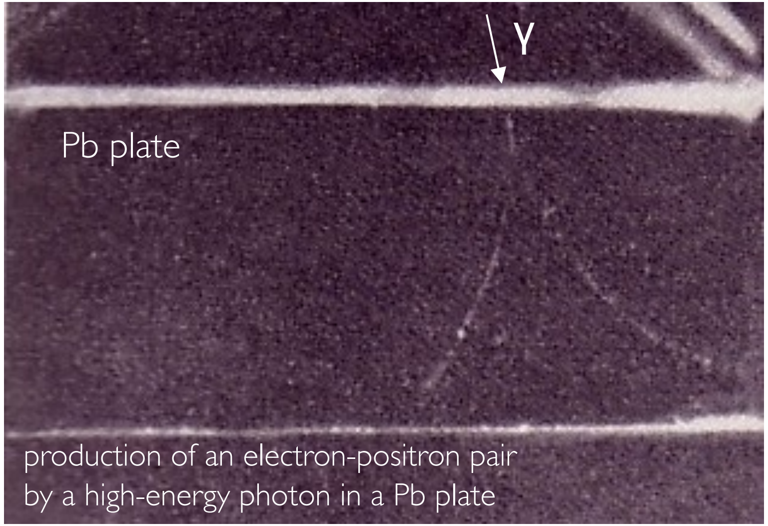
- Dominating process at high γ energies (>10 MeV)



Pair production in the field of the nucleus



Pair production in the field of an electron (strongly suppressed: probability ~ 1/Z)



production of an electron-positron pair by a high-energy photon in a Pb plate

Pair production

- Cross section for $E_\gamma >> m_e$
  - ✓ Accurate within a few percent down to energies as low as 1 GeV, particularly for high-Z materials

$$\sigma_{\mathrm{pair}} \simeq \frac{7}{9}\left(4\alpha r_e^2 Z^2 \ln\frac{183}{Z^{\frac{1}{3}}}\right)$$

$$\sigma_{\mathrm{pair}} \simeq \frac{7}{9}\frac{A}{N_A}\frac{1}{X_0}$$

Radiation length [g cm^{-2}]

$$X_0 = \frac{A}{N_A}\frac{1}{4\alpha r_e^2 Z^2 \ln\frac{183}{Z^{\frac{1}{3}}}}$$

The intensity of a photon beam traversing a block of material with thickness $X = 9/7\ X_0$ is reduced by a factor 1/e.

X_0 corresponds to the thickness of material where pair creation has a probability $P = 1 - e^{-7/9} \sim 54\%$

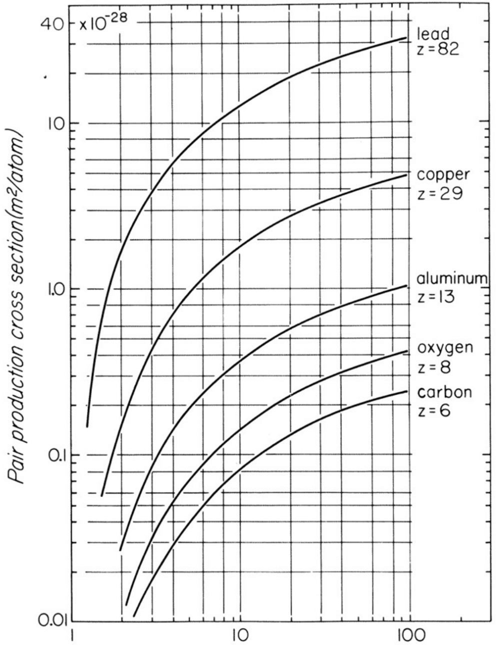
μ = mass attenuation coefficient [cm^2/g]

$$\mu = n\sigma = \frac{N_A}{A}\sigma_x$$

$$I(x) = I_0 e^{-\mu x}$$

$$\mu = \frac{7}{9}\frac{1}{X_0}$$

$$I(x) = I_0 e^{-\frac{7}{9}\frac{x}{X_0}}$$



Radiation Length

$$X_0 = \frac{A}{N_A}\frac{1}{4\alpha r_e^2 Z^2 \ln\frac{183}{Z^{\frac{1}{3}}}}$$

$$x_0 = \frac{X_0}{\rho}$$

Material	X_0 (g/cm^2)	ρ (g/cm^3)	x_0 (cm)
Pb			
Fe	13.84	7.87	1.76
Cu	12.86	8.96	1.44
Al	24.01	2.7	8.9
Air	36.66	1.205×10^{-3}	30 420
Water	36.08	1	36.1
H_2 (liq.)	63.05	0.0708	890
$PbWO_4$	7.39	8.28	0.89
BGO	7.97	7.13	1.12
LAr	19.55	1.396	14

Exercise: Radiation length values

- Three options
  1. Search tabulated value on PDG!
  2. Compute from full formula

$$X_0 = \frac{A}{4\alpha N_A \; Z^2 r_e^2 \; \ln \frac{183}{Z^{\frac{1}{3}}}}$$

  3. Compute from approximated formulas…

$$X_0 = 716.4 \text{ g cm}^{-2} \frac{A}{Z(Z+1)\ln \frac{287}{\sqrt{Z}}}$$

$$X_0 = 1433 \text{ g cm}^{-2} \frac{A}{Z(Z+1)(11.319 - \ln Z)}$$

$$X_0 = \frac{180A}{Z^2} \frac{\text{g}}{\text{cm}^2}$$

- What is the X_0 of Pb?

Exercise: Radiation length values

- Three options
  1. Search tabulated value on PDG!
  2. Compute from full formula

$$X_0 = \frac{A}{4\alpha N_A \, Z^2 r_e^2 \, \ln \frac{183}{Z^{\frac{1}{3}}}}$$

  3. Compute from approximated formulas…

$$X_0 = 716.4 \, \mathrm{g\,cm^{-2}} \frac{A}{Z(Z+1)\ln\frac{287}{\sqrt{Z}}}$$

X_0 = 6.3 g cm^{-2}

$$X_0 = 1433 \, \mathrm{g\,cm^{-2}} \frac{A}{Z(Z+1)(11.319 - \ln Z)}$$

X_0 = 6.3 g cm^{-2}

$$X_0 = \frac{180A}{Z^2} \frac{\mathrm{g}}{\mathrm{cm}^2}$$

X_0 = 5.5 g cm^{-2}\

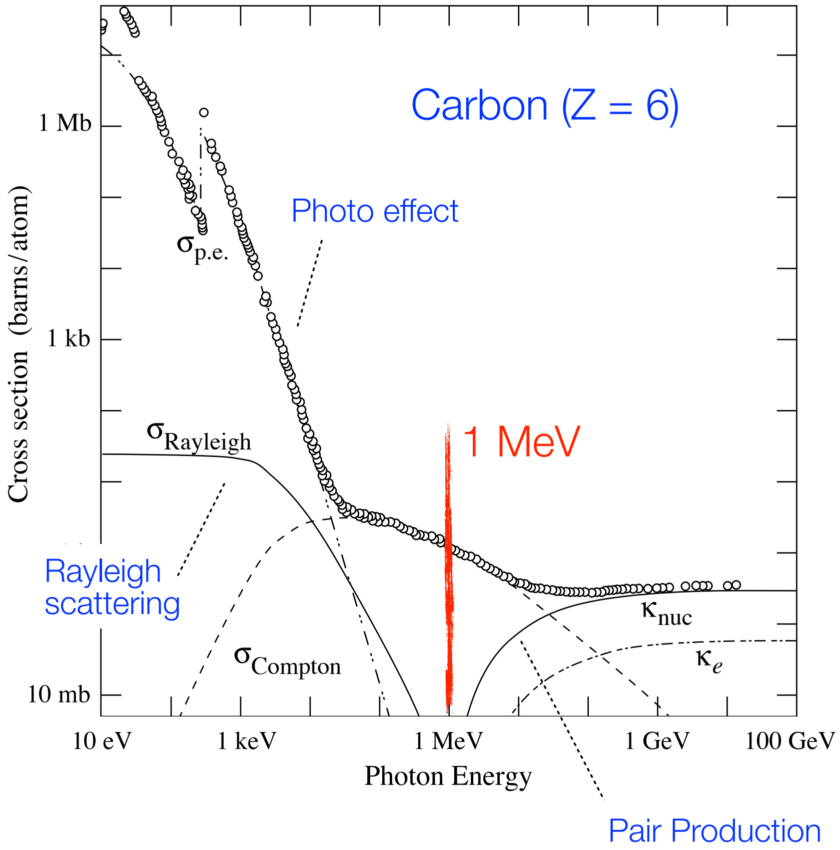
- What is the X_0 of Pb?
  - ✓ https://pdg.lbl.gov/2015/AtomicNuclearProperties/HTML/lead_Pb.html
  - ✓ A_{Pb} = 207
  - ✓ Z_{Pb} = 82

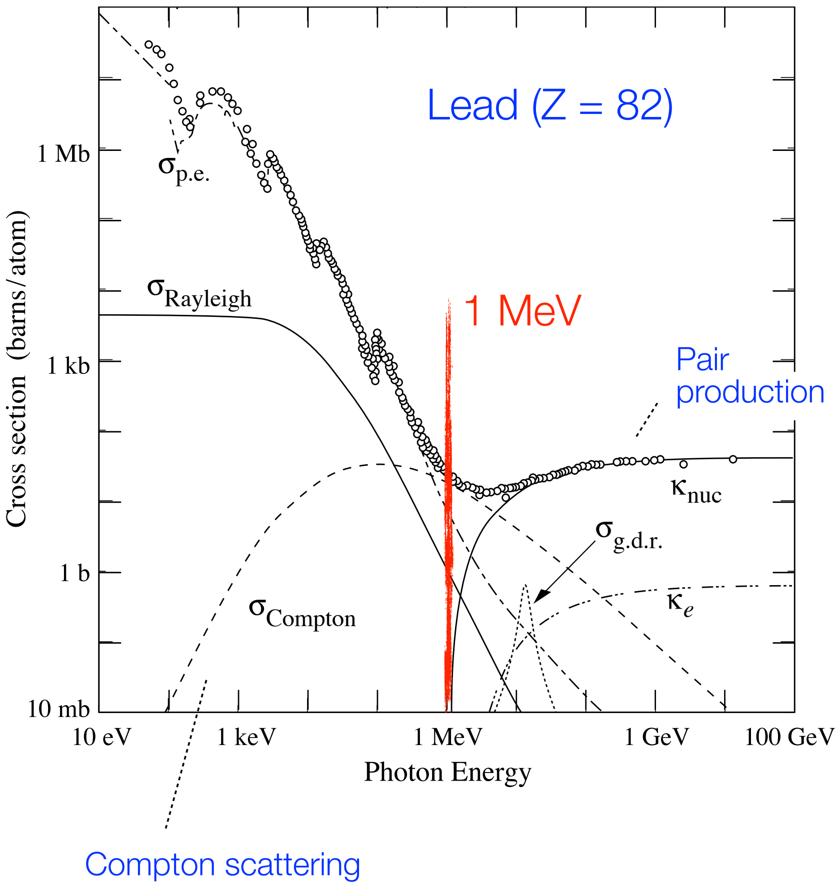
Exercise: Radiation length values

- Don't forget that to have a physical length one needs to consider the material density!

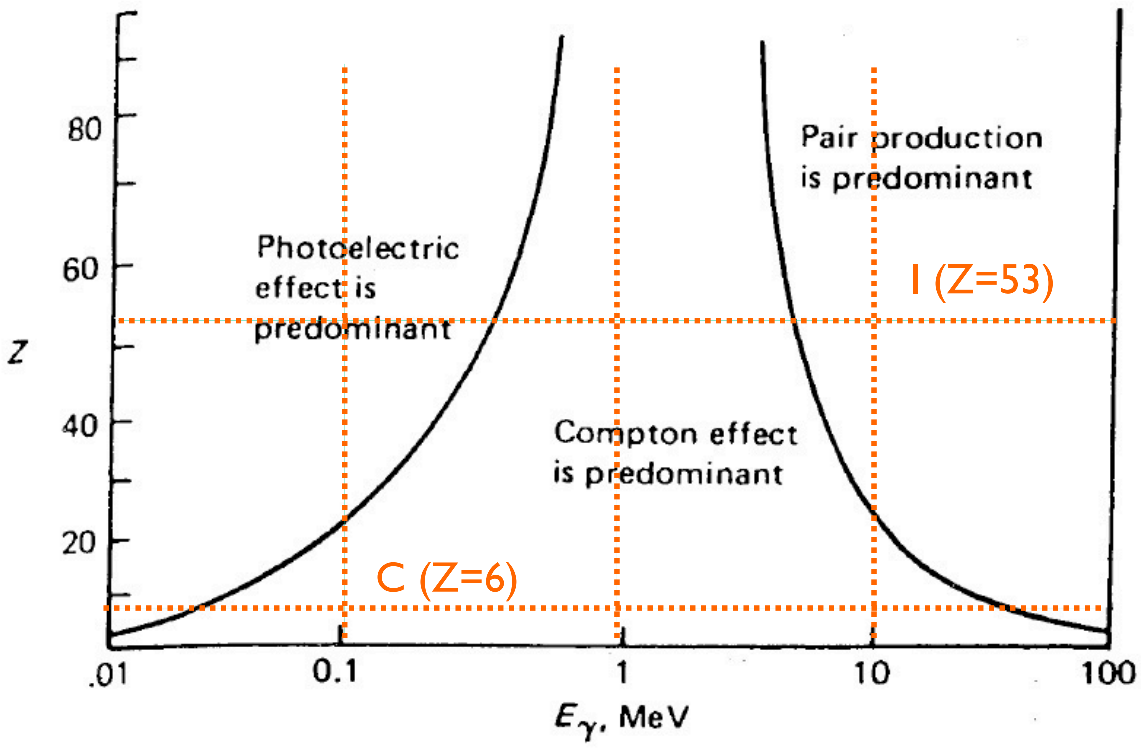
  - ✓ $X_0^{Pb} = 6.37\ \mathrm{g\ cm^{-2}}$
  - ✓ $\rho^{Pb} = 11.35\ \mathrm{g\ cm^{-3}}$
  - ✓ $x_0^{Pb} = X_0^{Pb} / \rho^{Pb} = 0.56\ \mathrm{cm}$

Recap: photon interaction total cross-section





Recap: photon interaction vs E and Z

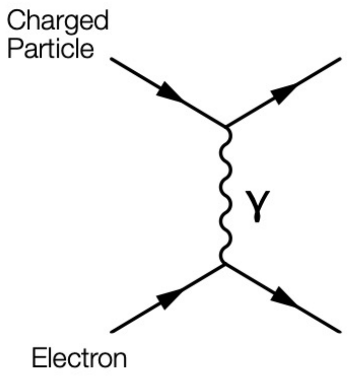


- E_γ = 0.1 MeV
  - ✓ C (Z=6) Compton dominant
  - ✓ I (Z=53) Photoelectric dominant
- E_γ = 1 MeV
  - ✓ Compton effect is dominant
- E_γ = 10 MeV
  - ✓ C (Z=6) Compton dominant
  - ✓ I (Z=53) pair production dominant

2.2

Charged-Particle Interactions in Matter

Energy loss by ionization: Bethe-Bloch formula



- Let's begin by assuming "heavy" particles ($M >> m_e$)
  - ✓ Interaction is dominated by elastic collision with matter electrons…
- Bethe-Bloch formula: mean energy loss per unit of *mass thickness*
  - ✓ Units: MeV g^{-1} cm^2 (multiply by density to get Mev/cm)

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

z = Charge of incident particle
M = Mass of incident particle
Z = Charge number of medium
A = Atomic mass of medium
I = Mean excitation energy of medium ~ 16 $Z^{0.9}$ eV
δ = Density correction [transverse extension of electric field]

$K = 4\pi N_A r_e^2 m_e c^2 = 0.307$ MeV g^{-1} cm^2

$T_{max} = 2m_ec^2\beta^2\gamma^2/(1 + 2\gamma\, m_e/M + (m_e/M)^2)$
[Max. energy transfer in single collision]

Bethe-Bloch: main dependencies

- Validity: $.05 < \beta\gamma < 500$, e.g. $M > m_\mu$

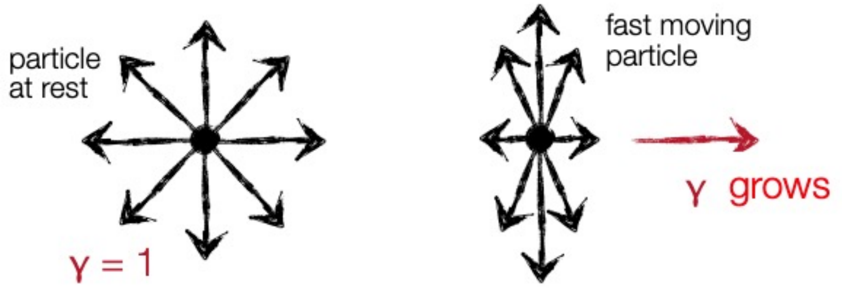
$$-\left\langle \frac{dE}{dx} \right\rangle \propto \frac{1}{\beta^2} \ln(\mathrm{const}\,\beta^2\gamma^2)$$

dE/dx falls ~ $1/\beta^2$ = kinematic factor

Slower particles feel electric force of atomic electrons for longer time

dE/dx rises ~ ln $(\beta\gamma)^2$ = relativistic rise

Relativistic extension of transversal E-field, relevant for $\beta\gamma > 4$. Transversal electric field of high-energy particle increases due to Lorentz transform; $E_y \rightarrow \gamma E_y$, interaction cross section increases

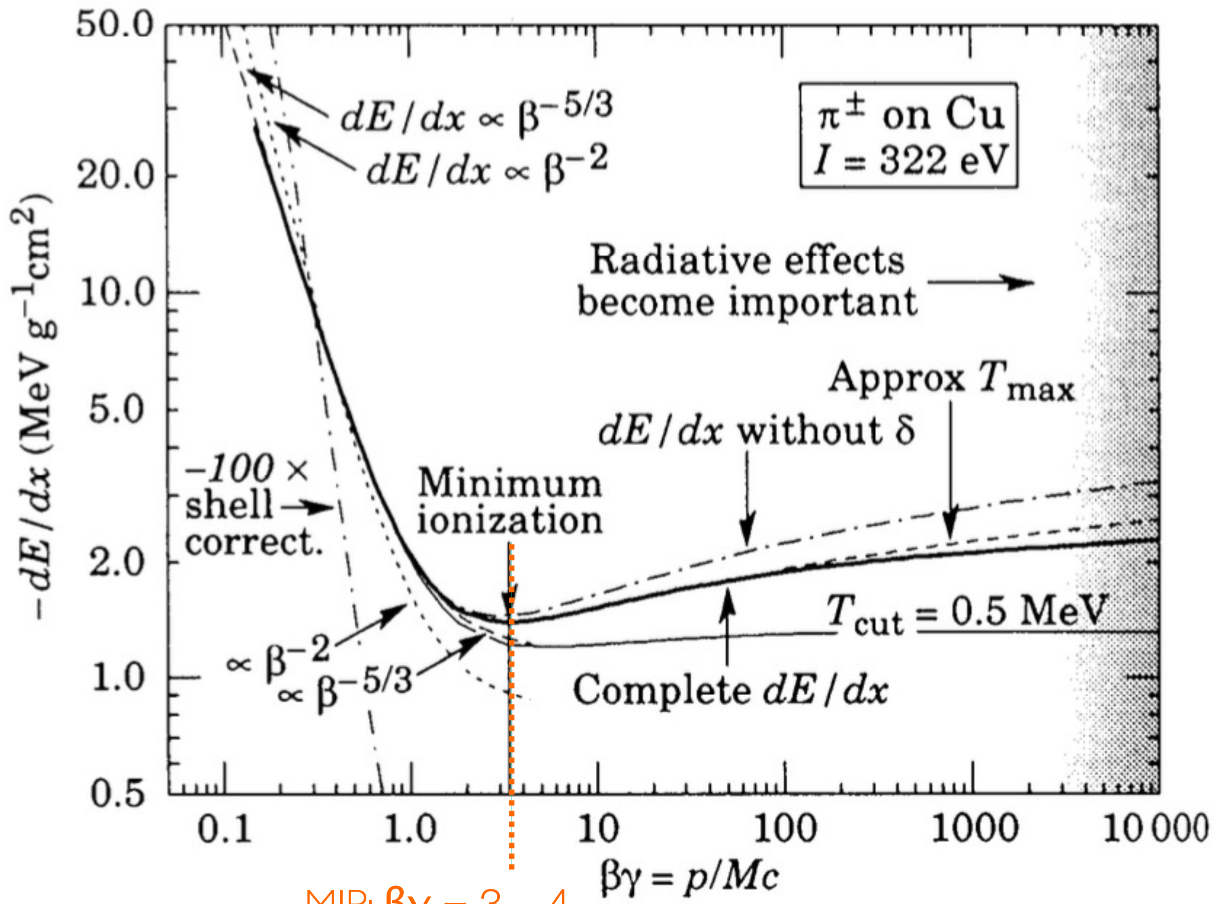


Corrections

- low energy: shell corrections
- high energy: density corrections (polarization of medium generates saturation at large $\beta\gamma$, parameterized by factor δ)

Bethe-Bloch example: pions on Copper

Units:
MeV g^{-1} cm^2



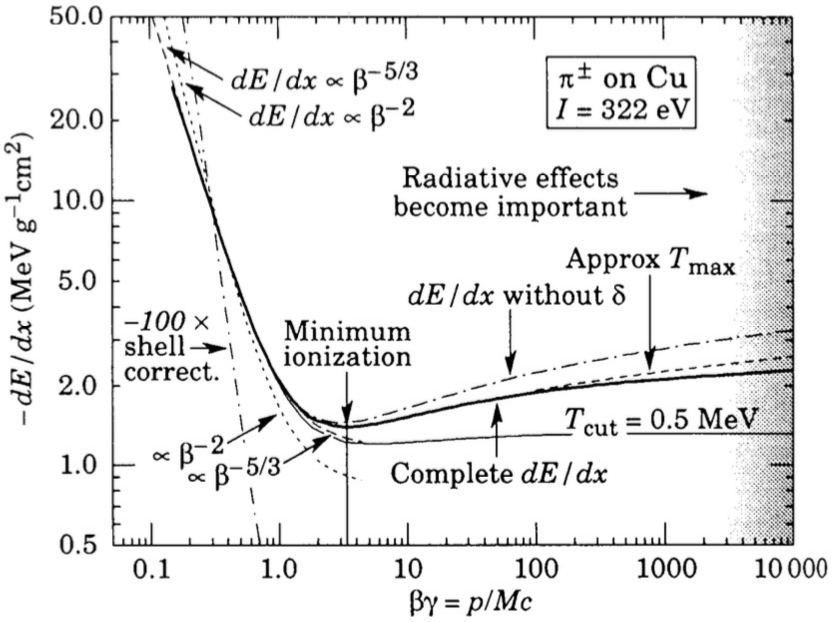
MIP: $\beta\gamma$ = 3 – 4

rho_Cu = 8.94 g/cm^3

MIP looses
~ 13 MeV/cm

Bethe-Bloch: low and high energy corrections

- Shell correction (low energy)
  - ✓ Arises if particle velocity is close to orbital velocity of electrons, i.e. $\beta c \sim v_e$
  - ✓ Assumption that electron is at rest breaks down
  - ✓ Capture process is possible
  - ✓ Shell correction are in general small
- Density correction (high energy)
  - ✓ Polarization effect, density dependent
  - ✓ Shielding of electrical field far from particle path
  - ✓ Effectively cuts of the long range contribution
  - ✓ Leads to saturation of <dE/dx>
  - ✓ More relevant at high γ
    - e.g. increased range of electric field, …



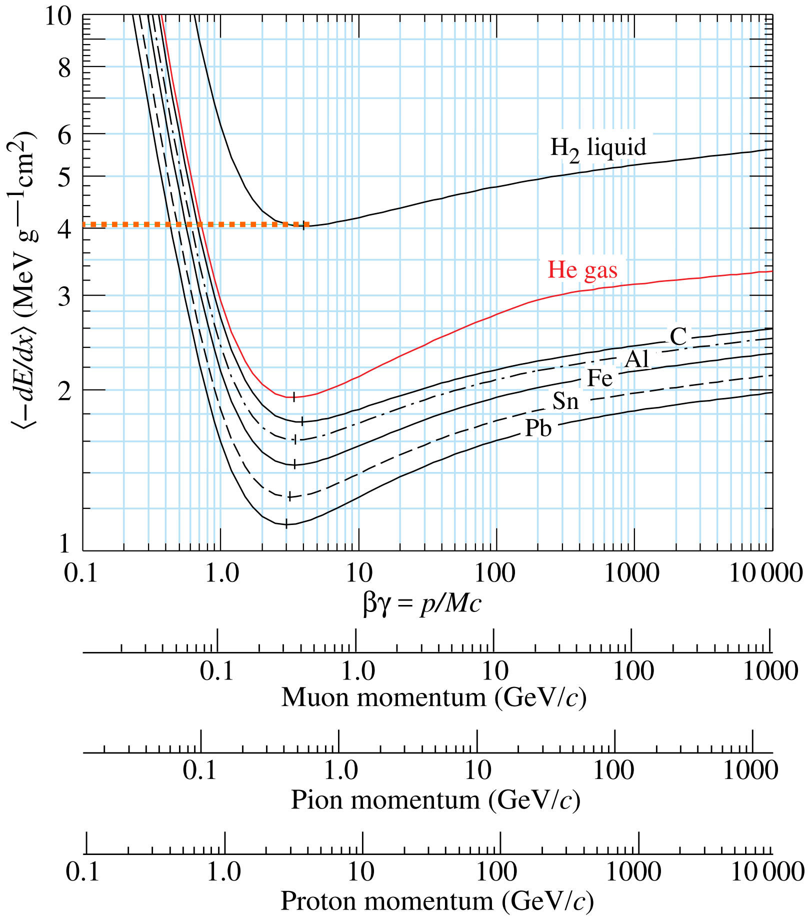
Energy loss dependence on material properties

$$-\left\langle \frac{dE}{dx} \right\rangle \propto \frac{Z}{A}$$

- A = mass of target nucleus
- Z = charge of target nucleus

Minimum ionization

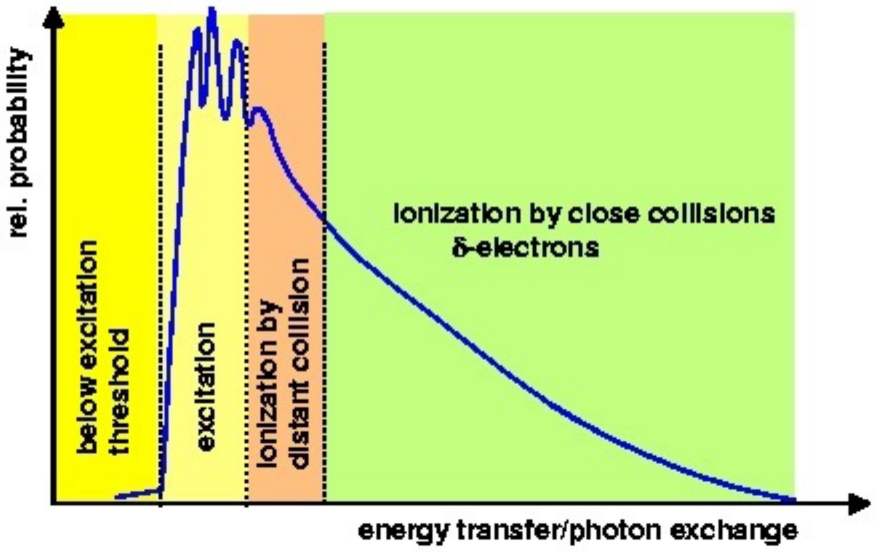
- ✓ ~ 1-4 MeV/g cm^{-2}
- ✓ Example: H_2 → 4 MeV/g cm^{-2}



Energy loss straggling: Landau distribution

- Bethe-Bloch formula gives average energy loss, but there can be large fluctuations
  - ✓ Energy loss ΔE in material of thickness Δx can be imagined as sum of energy losses δE_i in series of collisions
  - ✓ Energy loss δE_i distribute statistically
  - ✓ Landau distribution: probability distribution of energy loss in thin absorbers
    - Asymmetric: long high-energy tail from rare hard collisions Most probable value (MPV) < mean; the mean is ill-defined for very thin absorbers
    - Width parameter ξ sets the scale of the asymmetric tail of Landau distribution: long high-energy-loss tail caused by rare hard collisions (δ-rays)

$$\Delta E = \sum_i \delta E_i$$

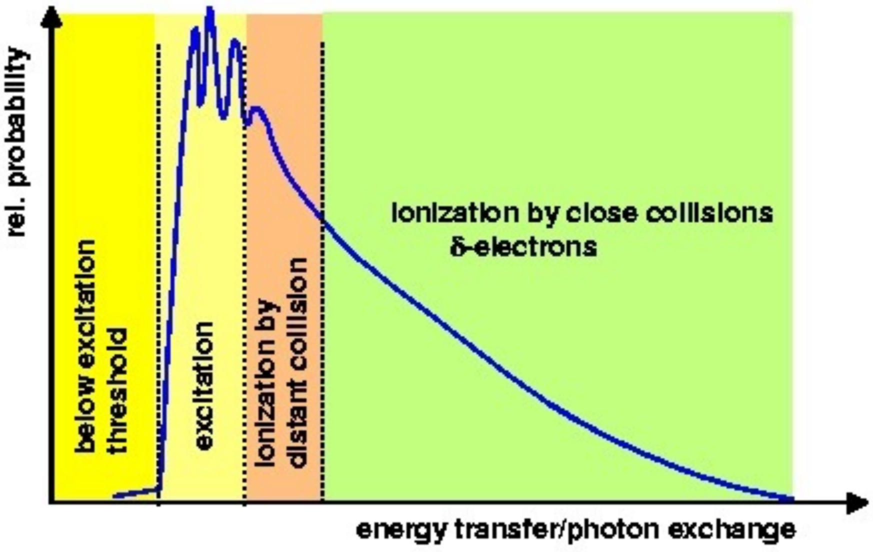


$$\delta E_i$$

Energy loss straggling: Landau distribution

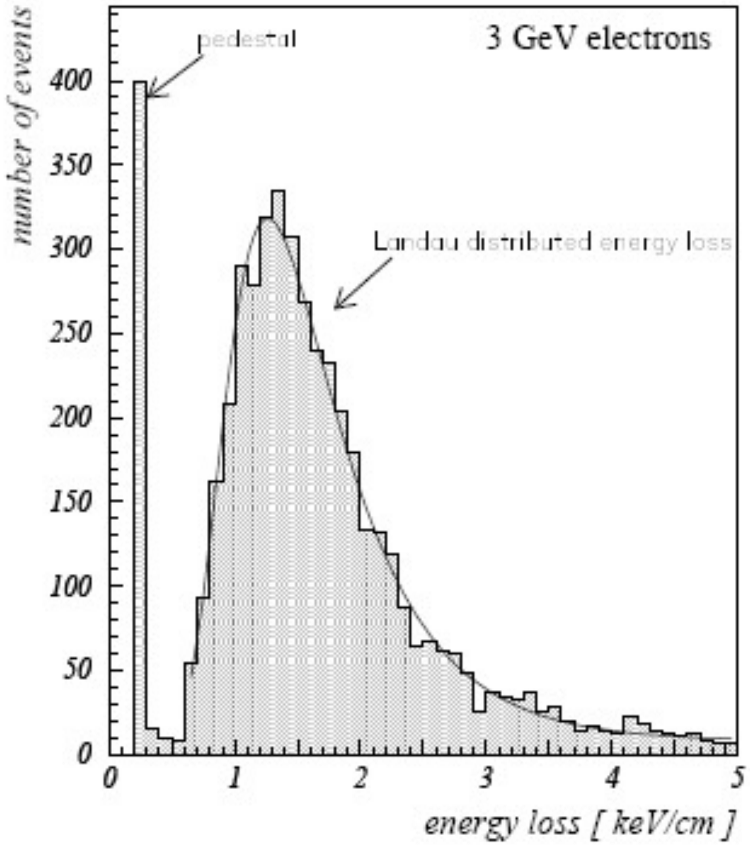
$$f(\lambda) = \frac{1}{2\pi} e^{-\frac{1}{2}(\lambda + e^{-\lambda})}$$

$$\lambda = \frac{\delta E - \delta \bar{E}}{\xi} \qquad \xi = \frac{K}{2} \frac{Z}{A} \frac{\rho x}{\beta^2}$$



- Width parameter ξ sets the scale of the asymmetric tail of Landau distribution: long high-energy-loss tail caused by rare hard collisions (δ-rays)
  - ✓ $K = 4\pi N_A r_e^2 m_e c^2 = 0.307$ MeV mol^{-1} cm^2
- **Practical consequence: MIP signal in a thin detector is Landau-distributed!**
  - ✓ Important for thin active layers in sampling calorimeters

Landau distribution: an example



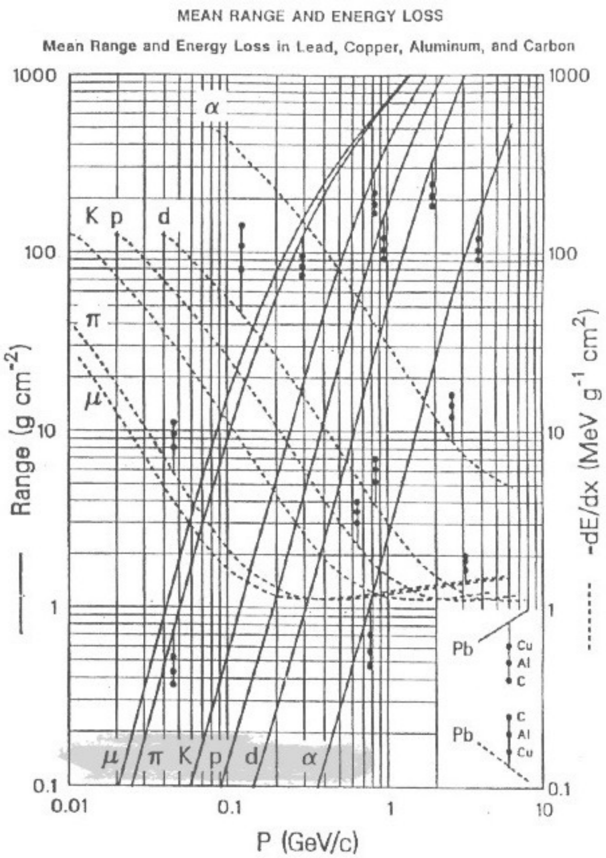
3 GeV electron in thin gap multi wired chamber

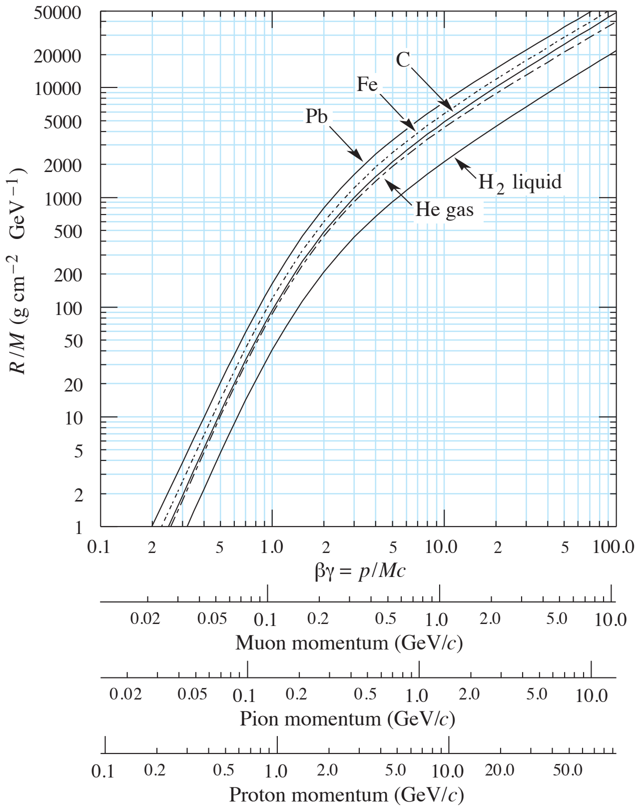
Mean energy range

- Integrate over energy loss from E to 0

$$R = \int_E^0 \frac{dE}{dE/dx}$$

- Example
  - ✓ Proton with p = 1 GeV
  - ✓ Target Pb (ρ = 11.35 g cm^{-2})
  - ✓ R = 200 g cm^{-2}
  - ✓ R[cm] = R/ρ ~ 18 cm





Energy loss of electrons

- Bethe-Bloch formula needs modification
  - ✓ Incident and target particles have same mass m_e
  - ✓ Scattering of identical, undistinguishable particles
    - Note that energy loss of positron at low energy has (slightly) different behavior as positrons are not identical with electrons…

$$-\left\langle \frac{dE}{dx} \right\rangle_{\mathrm{el.}} = K \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{m_e \beta^2 c^2 \gamma^2 T}{2I^2} + F(\gamma) \right]$$

T = kinetic energy of electron

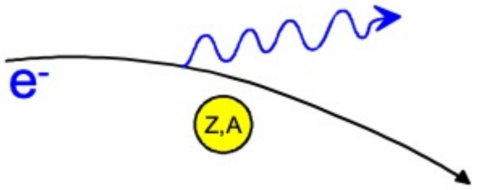
Energy loss of electrons vs heavy charged particles

- At high energy (β~1) energy loss for both heavy charged particles and electrons/positrons can be approximated
  - ✓ B factor parameterizes smaller rate of relativistic rise for electrons than for heavy particle (can be used to distinguish charged particles of different masses… but not in calorimeter!)

$$-\left\langle \frac{dE}{dx} \right\rangle \propto 2\ln\frac{2m_e c^2}{I} + A\ln\gamma - B$$

	A	B
electrons	3	1.95
heavy charged particles	4	2

Bremsstrahlung and Radiation Length



- Bremsstrahlung ("braking radiation") arises if particles are accelerated in Coulomb field of nucleus
  - ✓ Depend on m^{-2} → most important for "light" particles (i.e. electrons)
  - ✓ … or ultra-relativistic ones (e.g. muons)

$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ln \frac{183}{Z^{1/3}} \propto \frac{E}{m^2}$$

- Considering electrons…

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ln \frac{183}{Z^{1/3}}$$

$$X_0 = \frac{A}{N_A} \frac{1}{4\alpha r_e^2 Z^2 \ln \frac{183}{Z^{\frac{1}{3}}}}$$

Radiation length

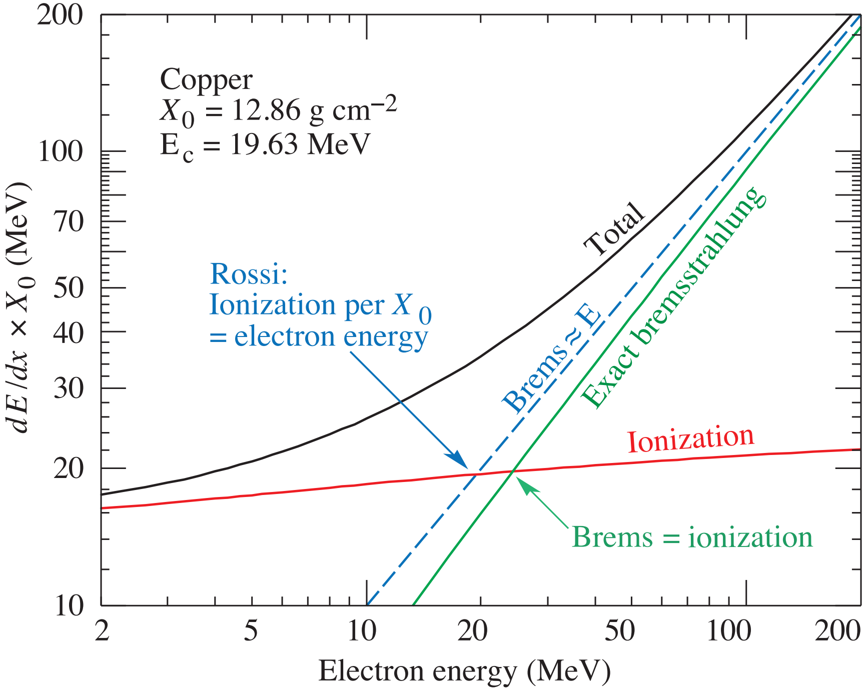
$$\frac{dE}{dx} = \frac{E}{X_0} \qquad E = E_0 \, e^{-\frac{x}{X_0}}$$

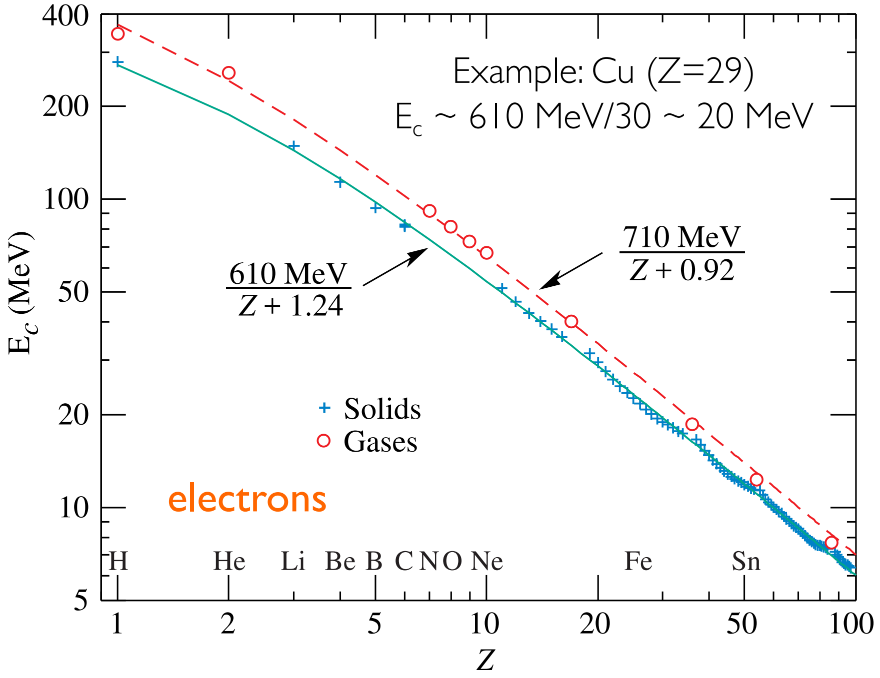
After passage of one X_0 electron has lost all but (1/e)th of its energy (e.g. 63%)

Critical energy

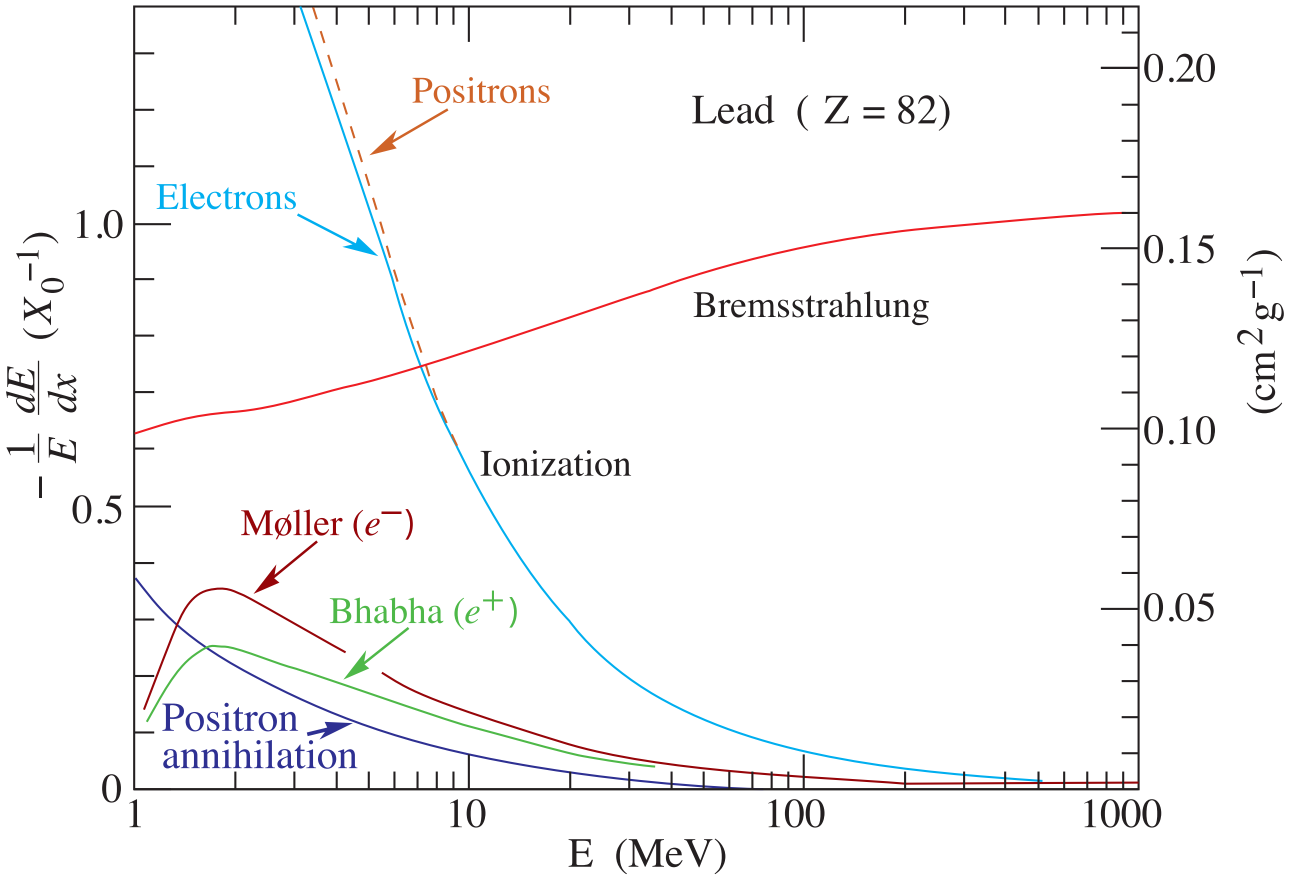
$$\left(\frac{dE}{dx}\right)_{\mathrm{Tot}} = \left(\frac{dE}{dx}\right)_{\mathrm{Ion}} + \left(\frac{dE}{dx}\right)_{\mathrm{Brems}}$$

$$\frac{dE}{dx}(E_c)\Bigg|_{\mathrm{Brems}} = \frac{dE}{dx}(E_c)\Bigg|_{\mathrm{Ion}}$$



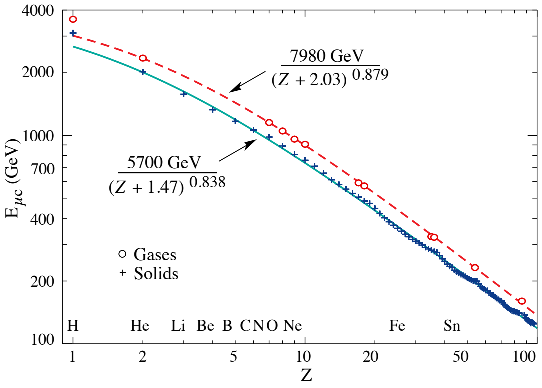


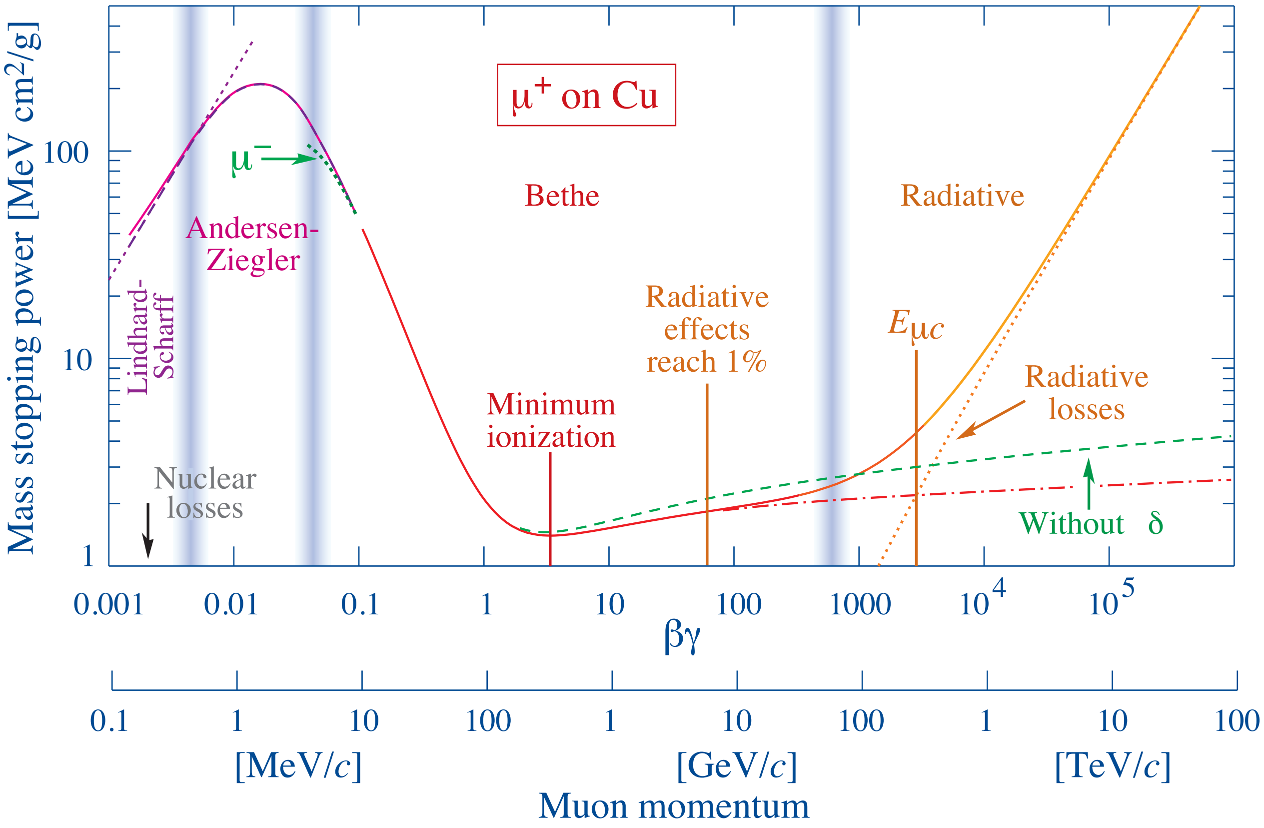
Recap: (fractional) energy loss by electrons



Muon energy loss

- Muon energy loss = Bethe-Bloch for βγ > 1
- Critical energy for muons $E_{c\mu}$ ~ O(100) GeV vs. few MeV for e
- In general, muons are minimum-ionizing through whole calorimeter
  - ✓ But for extremely high energies (e.g. future colliders) one could do muon calorimetry

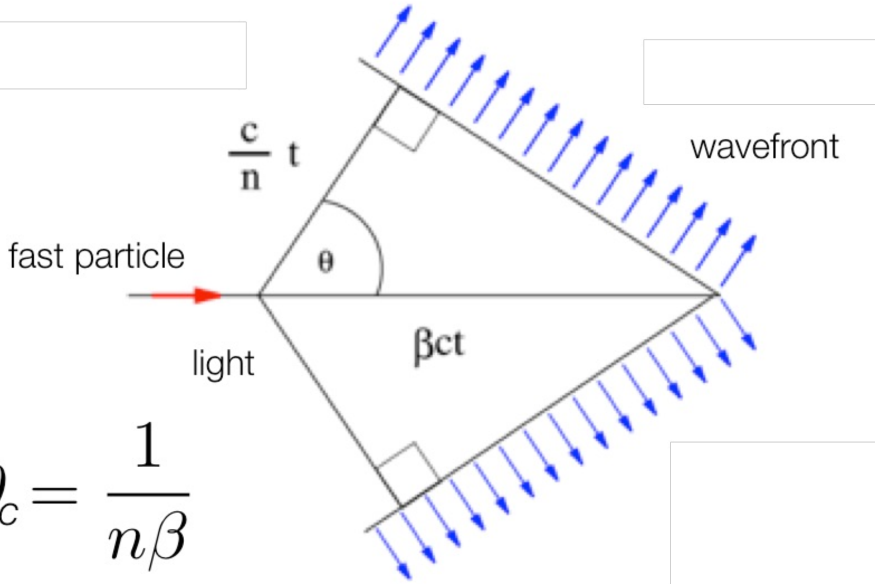




Cherenkov radiation

- Particles moving in a medium with speed larger than speed of light in that medium will lose energy by emitting electromagnetic radiation
  - ✓ Charged particle polarize medium generating an electrical dipole varying in time
  - ✓ Every point in trajectory emits a spherical EM wave, waves constructively interfere…

Why Cherenkov in a calorimetry course? Relevant for dual readout calorimetry (scintillation vs. Cherenkov in crystals)



$$\cos\theta_c = \frac{1}{n\beta}$$

$$\beta > \frac{1}{n}$$

refractive index $n = \sqrt{\varepsilon}$ dielectric constant

Cherenkov radiation

Medium	n	β_{thr}	θ_{max} [β=1]	N_{ph} [eV^{-1} cm^{-1}]
Air	1.000283	0.9997	1.36°	0.208
Isobutane	1.00127	0.9987	2.89°	0.941
Water	1.33	0.752	41.2°	160.8
Quartz	1.46	0.685	46.7°	196.4

$$\frac{d^2N}{dx\,dE} = \alpha z^2 \sin^2\theta_C \approx 370 \sin^2\theta_C \ \mathrm{eV}^{-1}\mathrm{cm}^{-1}$$

- Emitted photons are in optical region
  - ✓ E = 1-5 eV; λ = 300-600 nm, typically blue
- Energy loss by Cherenkov radiation very small
  - ✓ < 1% of ionization loss for Z > 7
  - ✓ 5% for light gas

Example: proton with E_{kin} = 1 GeV passing through 1 cm water

$\beta = p/E \sim 0.875$; $\cos\theta_C = 1/n\beta = 0.859 \rightarrow \theta_C = 30.8°$
$d^2N/(dEdx) = 370 \sin^2\theta_C \ eV^{-1} cm^{-1} \sim 100 \ eV^{-1} cm^{-1}$

$\Delta E_{loss} = \langle E \rangle \cdot d^2N/(dEdx) \cdot \Delta E \cdot \Delta x$
$\Delta E_{loss} = 2.5 \ eV \cdot 100 \ eV^{-1} cm^{-1} \cdot 5 \ eV \cdot 1 \ cm = 1.25 \ keV$

2.3

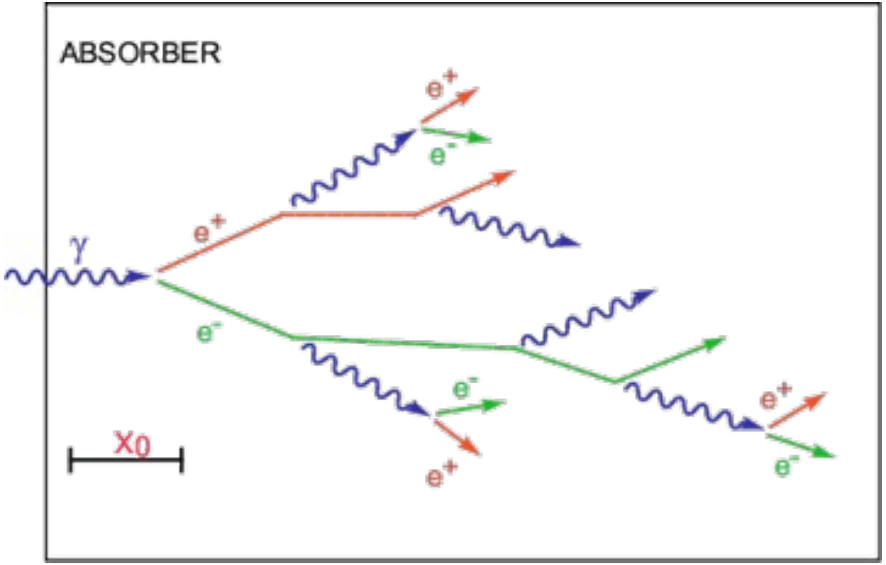
Electromagnetic Shower

Electromagnetic showers

- Dominant processes at high energy:
  - ✓ Photons: pair production
  - ✓ Electrons: bremsstrahlung

Radiation length [g cm^{-2}]

$$X_0 = \frac{A}{N_A} \frac{1}{4\alpha r_e^2 Z^2 \ln \frac{183}{Z^{\frac{1}{3}}}}$$



Pair production

$$\sigma_{\text{pair}} \simeq \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0}$$

$$I(x) = I_0 e^{-\frac{7}{9}\frac{x}{X_0}}$$

Bremsstrahlung

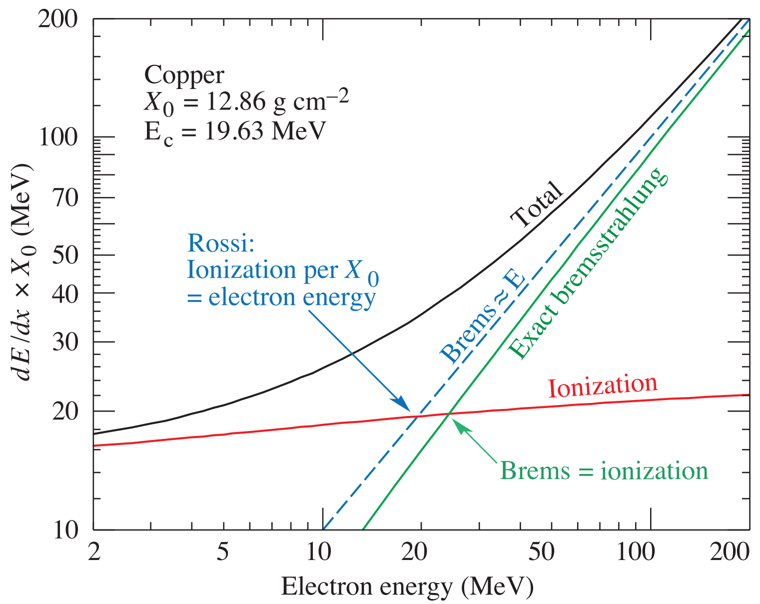
$$\frac{dE}{dx} = \frac{E}{X_0}$$

$$E = E_0\, e^{-\frac{x}{X_0}}$$

Electromagnetic showers

- Bremsstrahlung stops becoming dominant when E < E_C

$$\frac{dE}{dx}(E_c)\Big|_{\text{Brems}} = \frac{dE}{dx}(E_c)\Big|_{\text{Ion}}$$



$$E_c^{\text{Gas}} = \frac{710\text{ MeV}}{Z + 0.92}$$

$$E_c^{\text{Sol/Liq}} = \frac{610\text{ MeV}}{Z + 1.24}$$

Exercise: A simple shower model

Simple shower model:
[from Heitler]

Only two dominant interactions:
Pair production and Bremsstrahlung ...

γ + Nucleus ➛ Nucleus + e^+ + e^-
[Photons absorbed via pair production]

e + Nucleus ➛ Nucleus + e + γ
[Energy loss of electrons via Bremsstrahlung]

Shower development governed by X_0 ...

After a distance X_0 electrons remain with only $(1/e)^{th}$ of their primary energy ...

Photon produces e^+e^--pair after $9/7 X_0 \approx X_0$ ...

Assume:

$E > E_c$: no energy loss by ionization/excitation

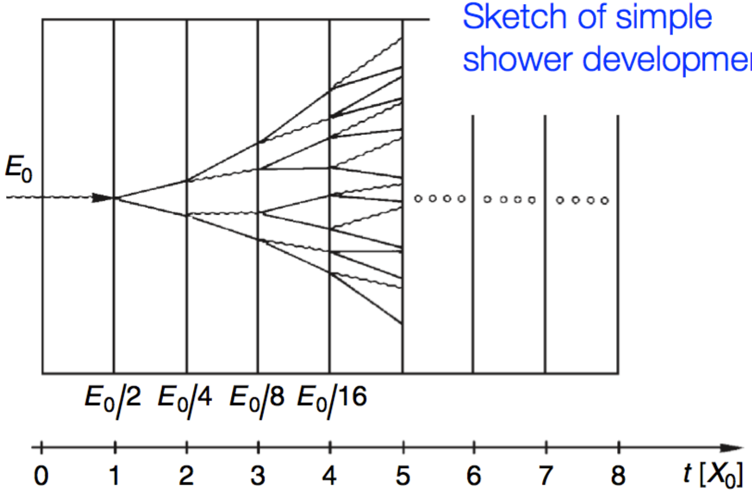
$E < E_c$: energy loss only via ionization/excitation

Use
Simplification:

$E_\gamma = E_e \approx E_0/2$
[E_e looses half the energy]

$E_e \approx E_0/2$
[Energy shared by e^+/e^-]

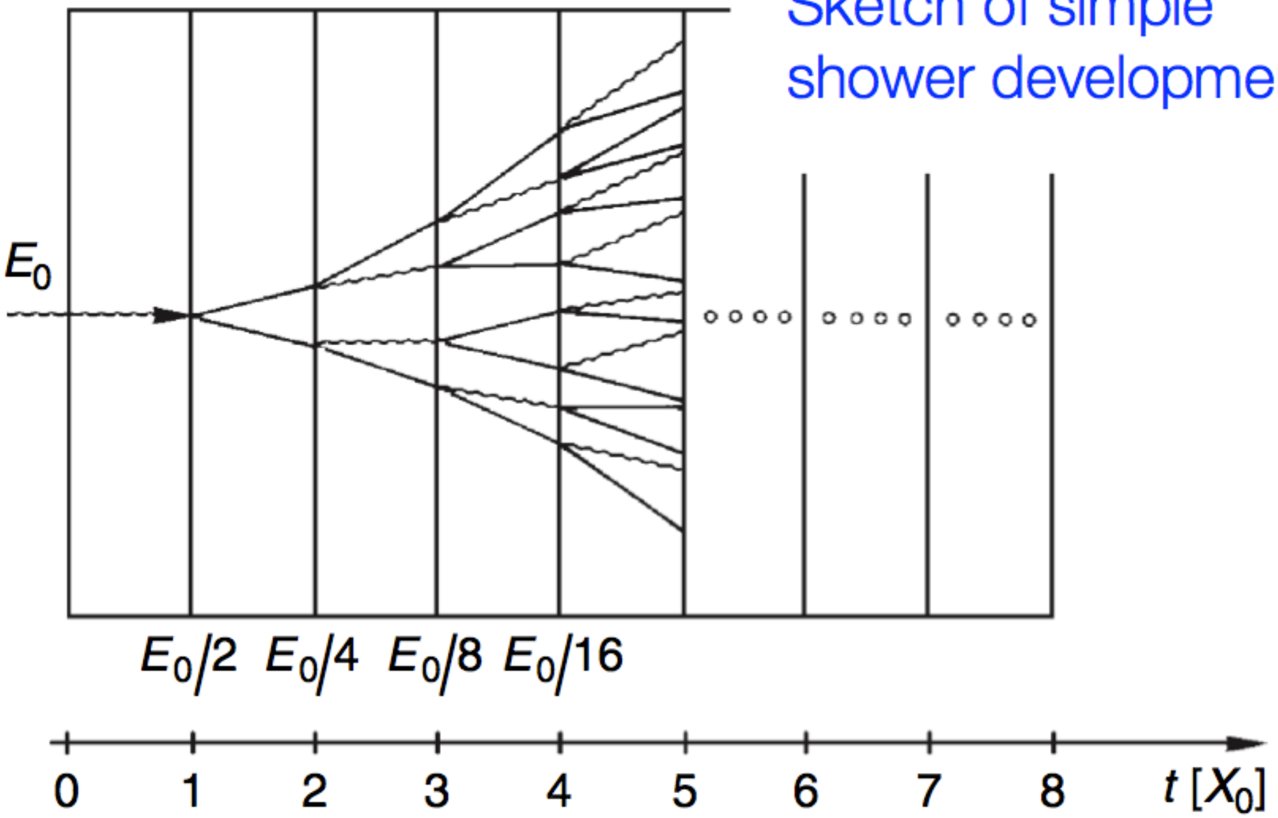
... with initial particle energy E_0



Sketch of simple shower development

- **Where does the shower reach its maximum?**

Exercise: A simple shower model



Sketch of simple shower development

Exercise: A simple shower model

Shower characterized by:

Number of particles in shower
Location of shower maximum
Longitudinal shower distribution
Transverse shower distribution

Longitudinal components; measured in radiation length ...

... use: $t = \frac{x}{X_0}$

Number of shower particles after depth t:

$$N(t) = 2^t$$

Energy per particle after depth t:

$$E = \frac{E_0}{N(t)} = E_0 \cdot 2^{-t}$$

$$\rightarrow \quad t = \log_2(E_0/E)$$

Total number of shower particles with energy E_1:

$$N(E_0, E_1) = 2^{t_1} = 2^{\log_2(E_0/E_1)} = \frac{E_0}{E_1}$$

Number of shower particles at shower maximum:

$$N(E_0, E_c) = N_{\max} = 2^{t_{\max}} = \frac{E_0}{E_c} \propto E_0$$

Shower maximum at:

$$t_{\max} \propto \ln(E_0/E_c)$$

Exercise: A simple shower model

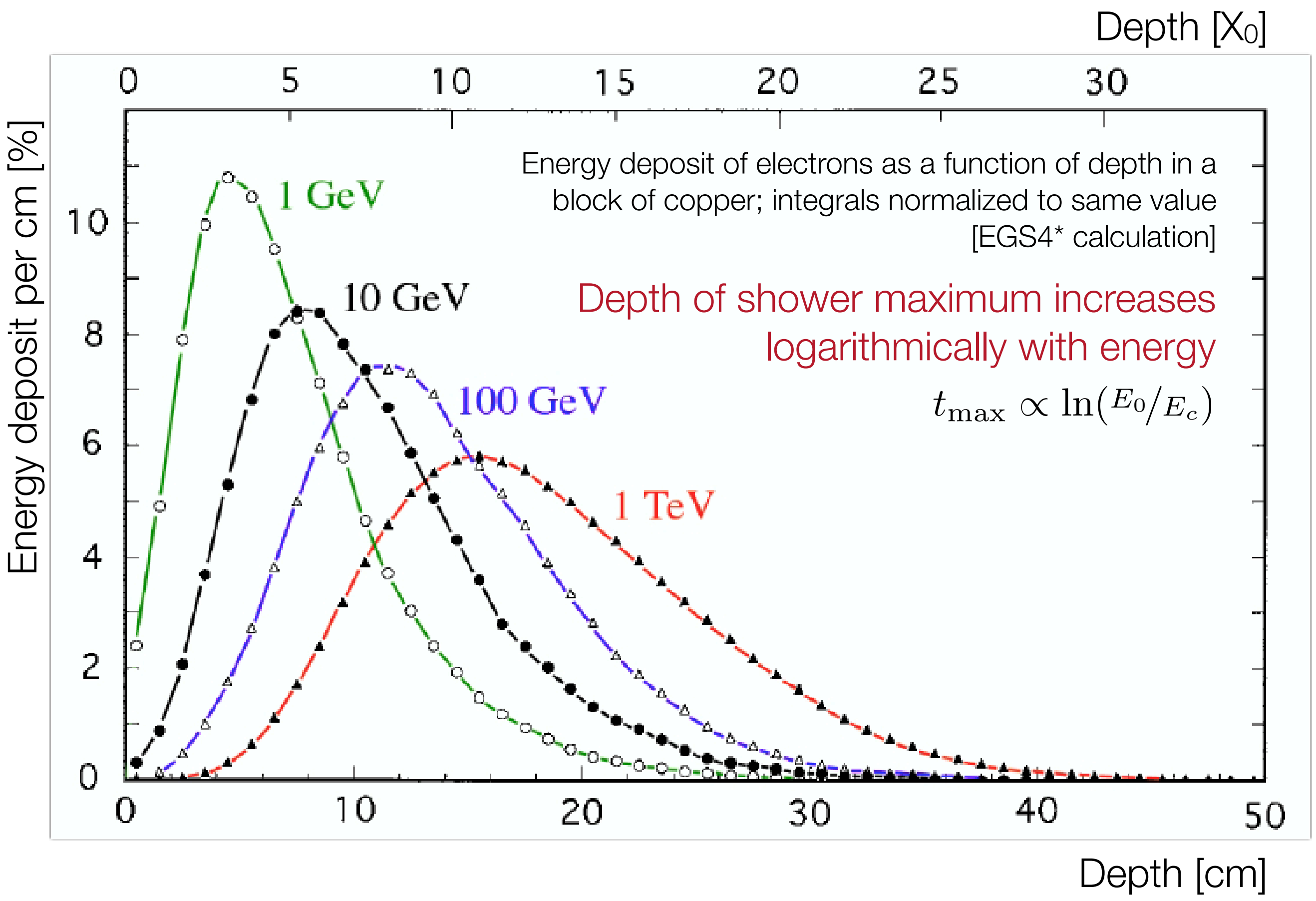
Simple shower model:
[continued]

Longitudinal shower distribution increases only logarithmically with the primary energy of the incident particle ...

Some numbers: $E_c \approx 10$ MeV, $E_0 = 1$ GeV ➛ $t_{max} = \ln 100 \approx 4.5$; $N_{max} = 100$
$E_0 = 100$ GeV ➛ $t_{max} = \ln 10000 \approx 9.2$; $N_{max} = 10000$

$$t_{\max}[X_0] \sim \ln \frac{E_0}{E_c}$$

EM shower longitudinal development



Limitations of the simple shower model

- Energy dependence of the cross-section
  - ✓ Simple shower abruptly stops at t_{max}

- Electrons undergo multiple Coulomb scattering, affecting the lateral spread

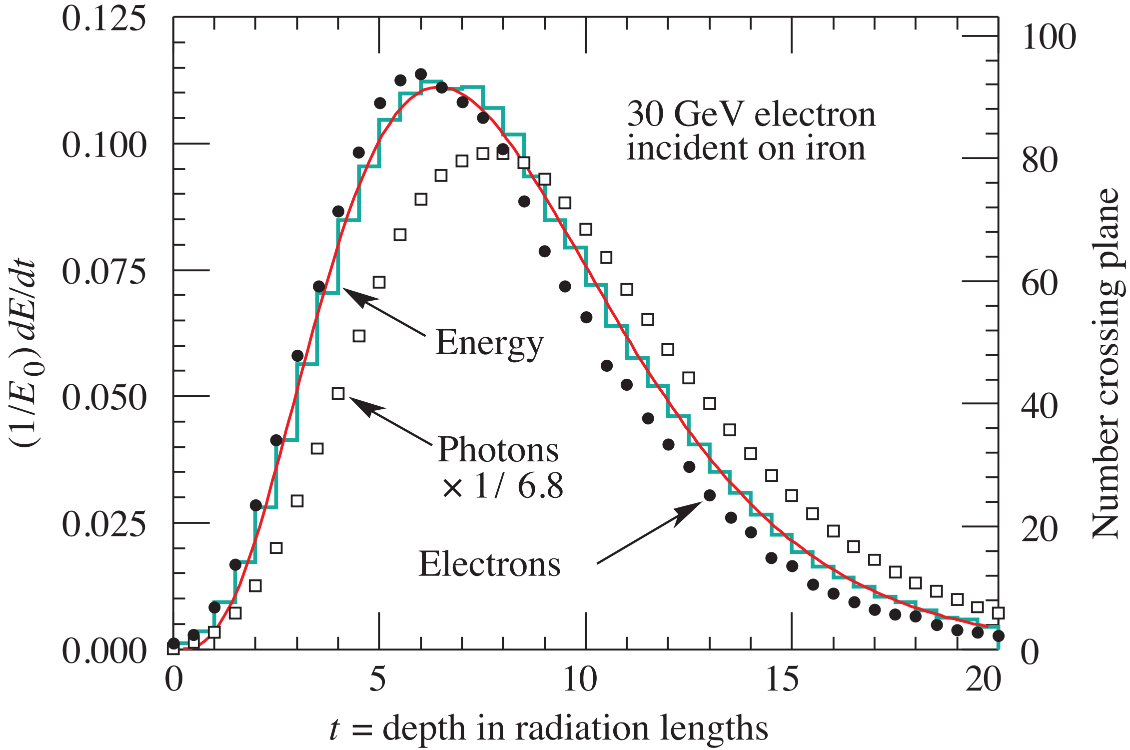
- Difference between showers induced by γ and electrons
  - ✓ $\lambda_{pair} = (9/7)\, X_0$

- Fluctuations: number of electrons/positrons produced not governed by Poisson statistics

- Some dependence on material makes development deviates from pure X_0 scaling

- Not all energy of electrons and positrons in the shower can be detected

EM shower: electrons vs photons



$$X_{\max} = X_0 \left(\ln \frac{E_0}{E_c} + C_{e\gamma} \right)$$

$C_e = 0.5$

$C_\gamma = -0.5$

EM shower longitudinal development parameterization

$$\frac{dE}{dt} = E_0\, b\, \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)}$$

- a determines shower max
- Shower maximum a/b - 1/b
- b *nearly* universal
  - ✓ b ~ 0.5 for e

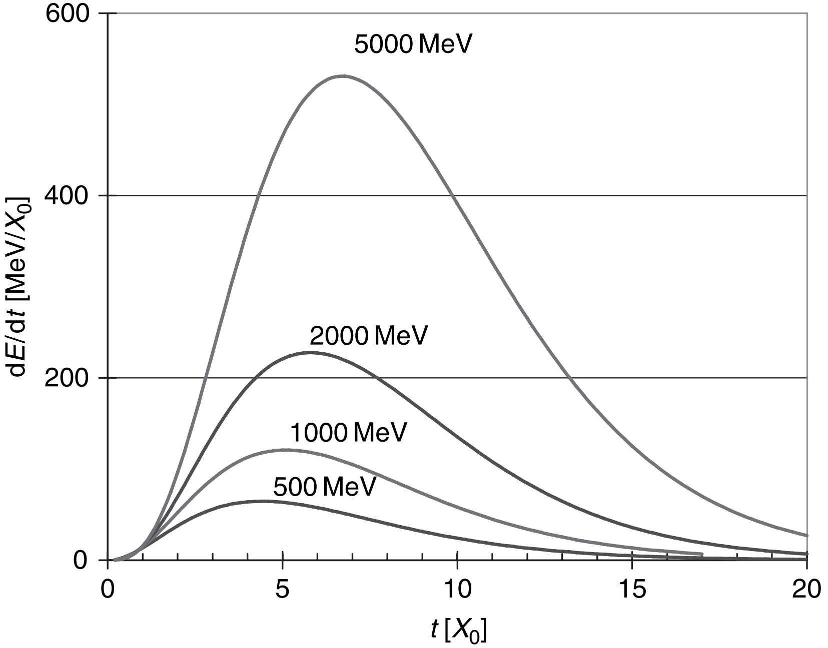
Detector design remark: with longitudinal segments (e.g. ATLAS) can sample shower profile

$$t = x/X_0 \qquad y = E/E_c$$

$$t_{\max} = (a-1)/b = 1.0 \times (\ln y + C_j)$$

$$j = e, \gamma$$

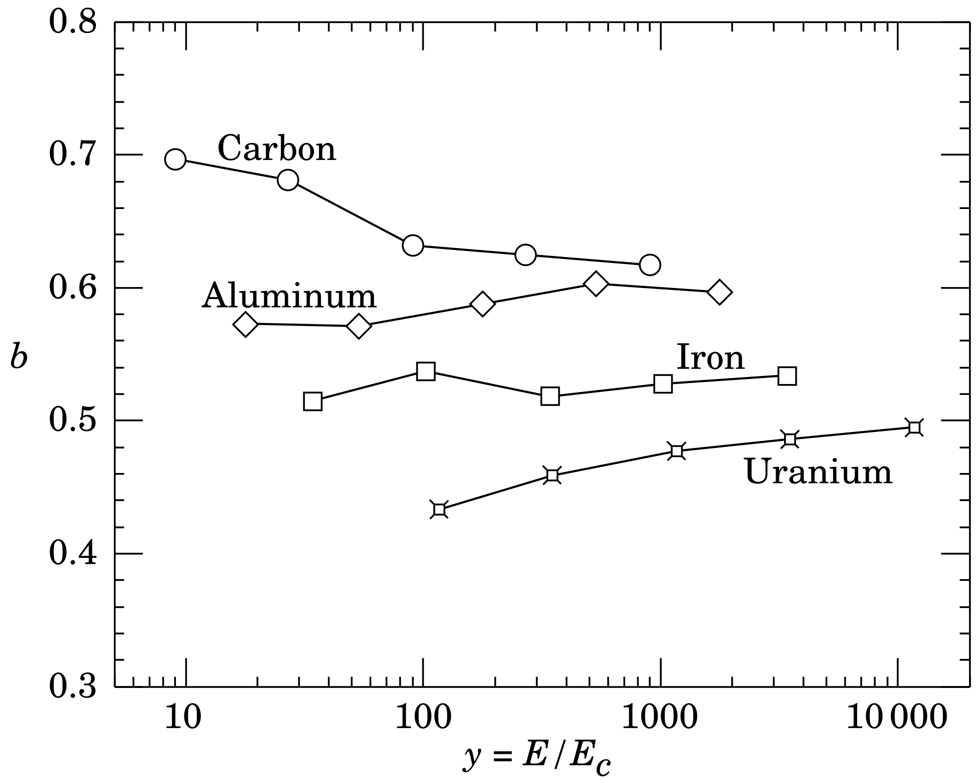
$X_s = X_0/b$ shower length



Longitudinal dependence on material

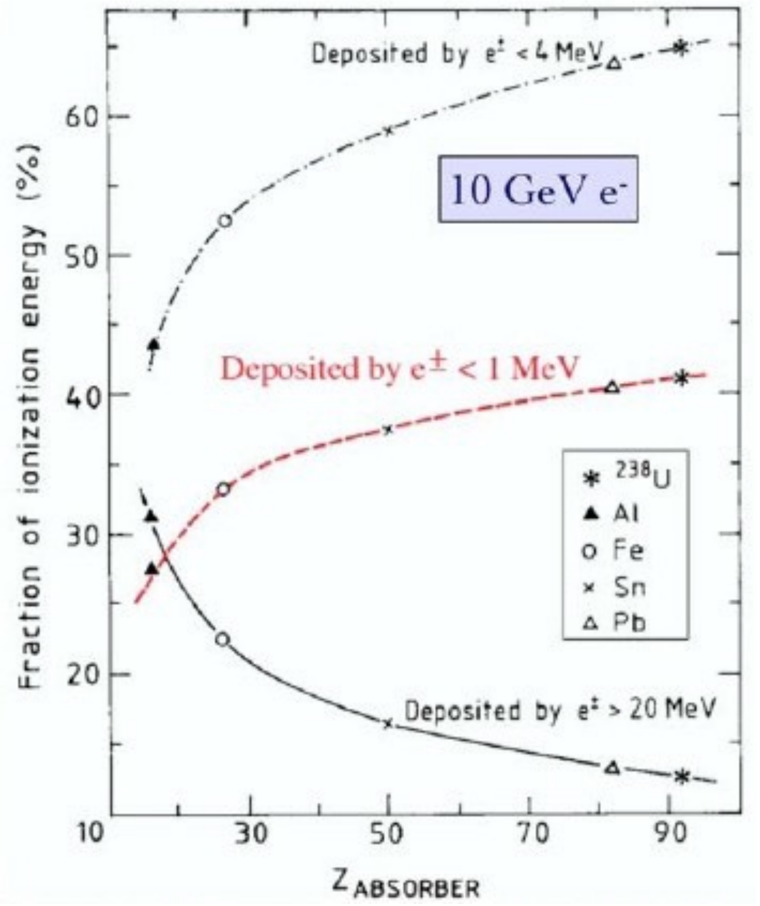
- b *nearly* universal…

$$X_s = X_0/b \quad \text{shower length}$$



Who deposits the EM shower energy?

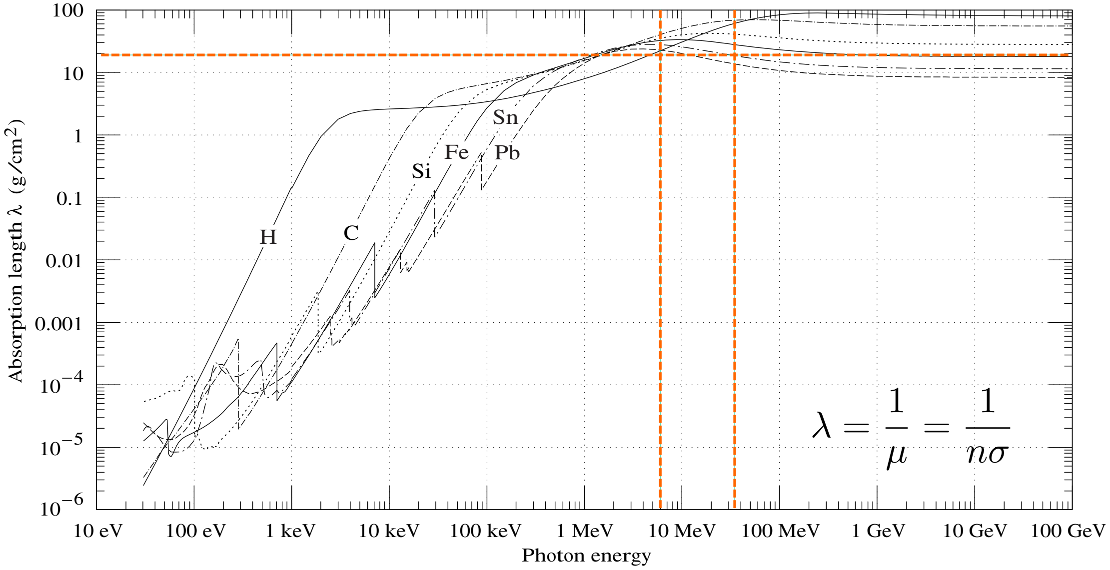
- Energy is deposited (in active medium) by low energy electrons/positrons and photons...
  - ✓ ~3/4 of the energy deposited by electrons ($e^{\pm}$): pair-production secondaries and Compton photoelectrons (~half of the electron component)
  - ✓ ~1/4 by photons below the pair-production threshold (photoelectric absorption)
    - *These are isotropic, have forgotten direction of incoming particle*
  - ✓ The typical shower particle is a 1 MeV electron, range < 1 mm
    - *Important consequences for sampling calorimetry*



Longitudinal dependence on material

- After shower maximum particle production stops
  - ✓ Electrons/positrons and photons have energies in ~5-20 MeV range (typical E_c size) of the
  - ✓ Electrons and positrons stop quickly in layer of ~ 1 X_0 (ionization loss)
  - ✓ Photons (~N_{max}/3) absorbed by photo-electric effect and/or Compton

- Absorption length: ~3λ needed to absorb 95% photons
- For 5-20 MeV photons λ ~ 20 g/cm² (approximately material independent)
- ~60 g/cm² after shower maximum
- Aluminum X_0 = 24 g/cm² (9 cm)
  - ✓ ~2.5 X_0 Al needed after t_{max} to absorb 95% of initial particle energy
- Pb X_0 = 6.4 g/cm² (0.56 cm)
  - ✓ ~9.3 X_0 Pb needed after t_{max} to absorb 95% of initial particle energy

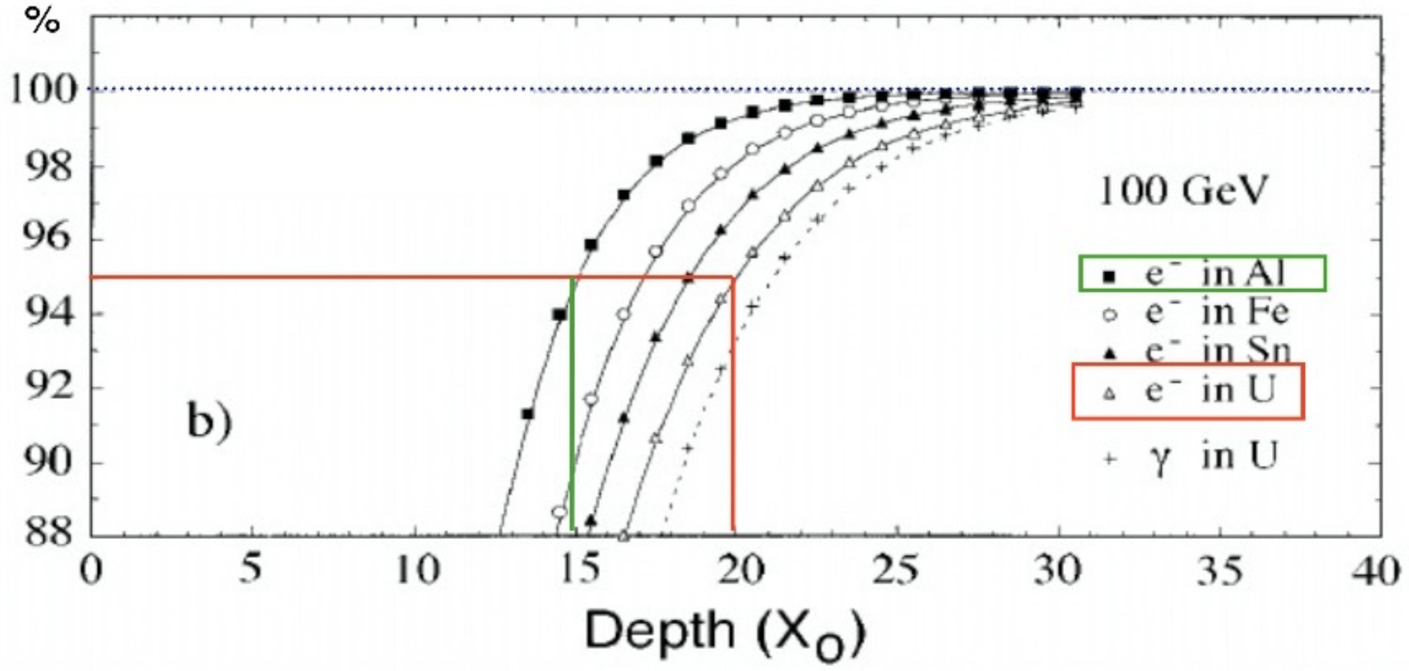


$$\lambda = \frac{1}{\mu} = \frac{1}{n\sigma}$$

More X_0 of Pb than Al will be needed after shower maximum to contain 95% of the shower energy!

Shower energy containment parametrisation

$$t_{\max} \simeq \ln \frac{E_0}{E_c} \qquad t_{95}[X_0] \simeq t_{\max} + 0.08Z + 9.6$$



Multiple scattering in EM showers

- MS model in EM shower = combination of single scattering part (high deflection regime) + Gaussian core (sum of many small-angle deflections)
  - ✓ 2% large scattering angles
  - ✓ 98% Gaussian around scattering angle 0

$$\langle \theta_k^2 \rangle = \sum_{m=1}^{k} \theta_m^2 = k \langle \theta^2 \rangle$$

$$\sqrt{\langle \theta^2 \rangle} \sim \frac{1}{\beta p} z \sqrt{\frac{x}{X_0}} \left(1 + k \ln \frac{x}{X_0} \right)$$

$\beta = 1$ $z = 1$ $k \sim 0.038$

- Strength of scattering ~ 1/p
  - ✓ Affect measurement of low momentum particles
  - ✓ Better to use light materials for compact showers

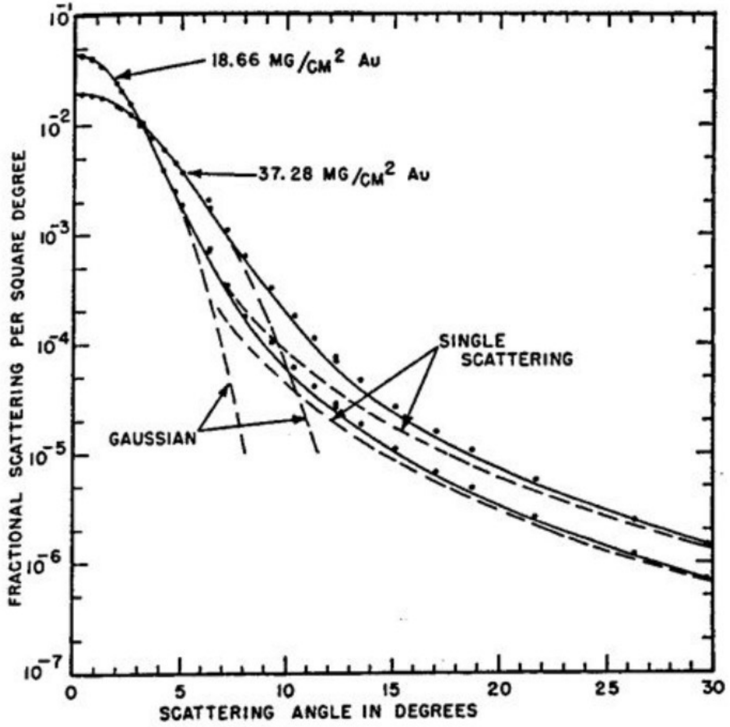
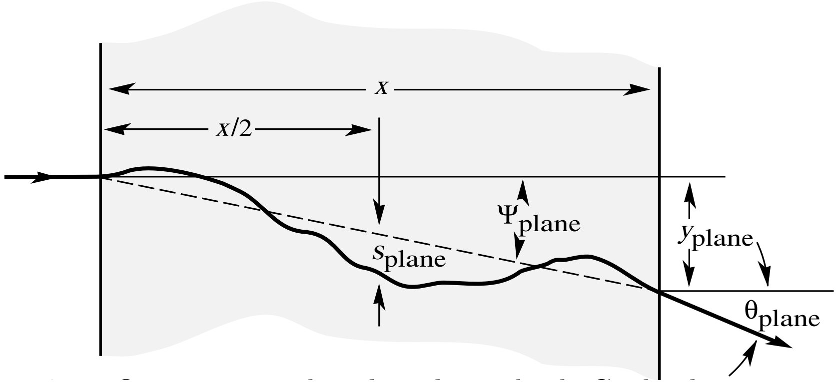


FIG. 3. Angular distribution of electrons from thick and thin gold foils from 0° to 30°. The solid line represents the theory of Molière extrapolated through the region where his small and large angle approximations give different values. The dotted lines at small angles represent the continuation of the gaussians of Fig. 1. At larger angles, the dotted line represents the single scattering contribution.

EM shower transverse development: Moliere radius



- High energy (early) part of the shower dominated by pair production (photons) and radiation (electrons)
  - ✓ Negligible opening angles

$$\langle\theta^2\rangle \sim \left(\frac{m}{E}\right)^2 = \frac{1}{\gamma^2}$$

- Shower transverse size driven by the low energy part of the shower
  - ✓ Dominant process: multiple scattering of low energy electrons

Multiple scattering angle distribution (Gaussian)

2d
$$\sqrt{\langle\theta^2\rangle} \sim \frac{13.6\,\mathrm{MeV}}{p}\sqrt{\frac{x}{X_0}}$$

3d extra factor √2
$$\sqrt{\langle\theta^2\rangle_{3d}} \sim \frac{19.2\,\mathrm{MeV}}{p}\sqrt{\frac{x}{X_0}}$$

Shower lateral profile estimate

Consider multiple scattering of e just after the shower maximum energies ~ E_c

Assume approximate range ~ 1 X_0

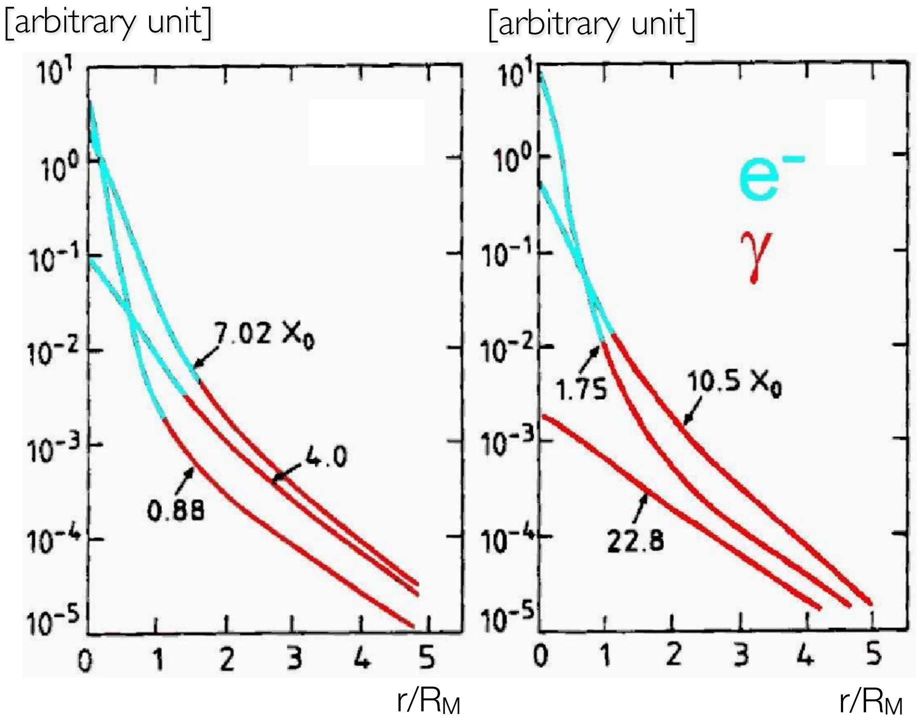
$$R = \langle\theta\rangle X_0$$

Containment
~90% @ R = 1 R_M
~95% @ R = 2 R_M
~99% @ R = 3.5 R_M

Moliere radius

$$R_M = \langle\theta\rangle_{x=X_0} \cdot X_0 \simeq \frac{21\,\mathrm{MeV}}{E_c} X_0$$

EM shower transverse profile parameterization



$$\frac{dE}{dr} = \alpha e^{-\frac{r}{R_M}} + \beta e^{-\frac{r}{\lambda_{min}}}$$

α, β : free parameters

R_M : Molière radius

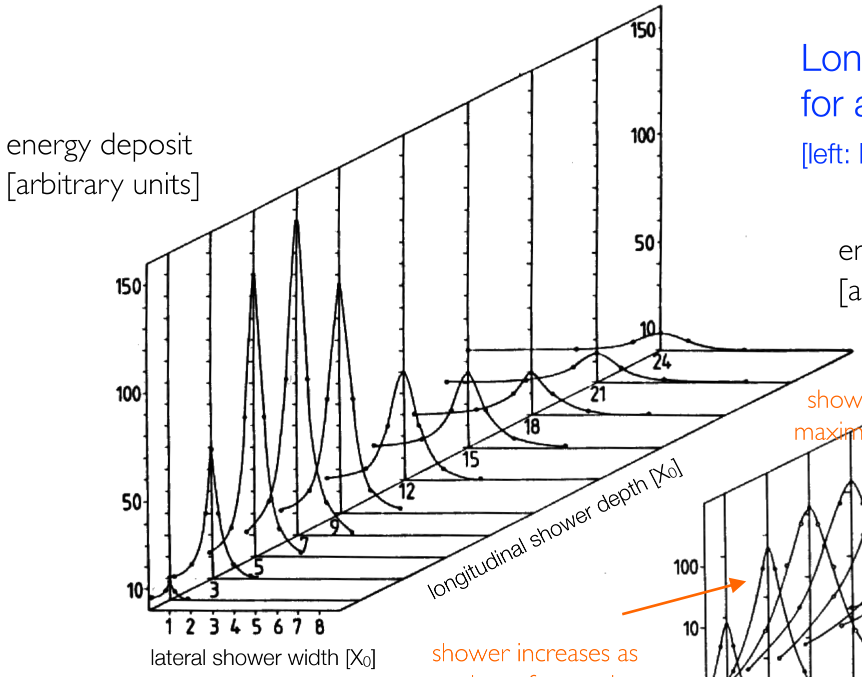
λ_{min} : range of low energetic photons

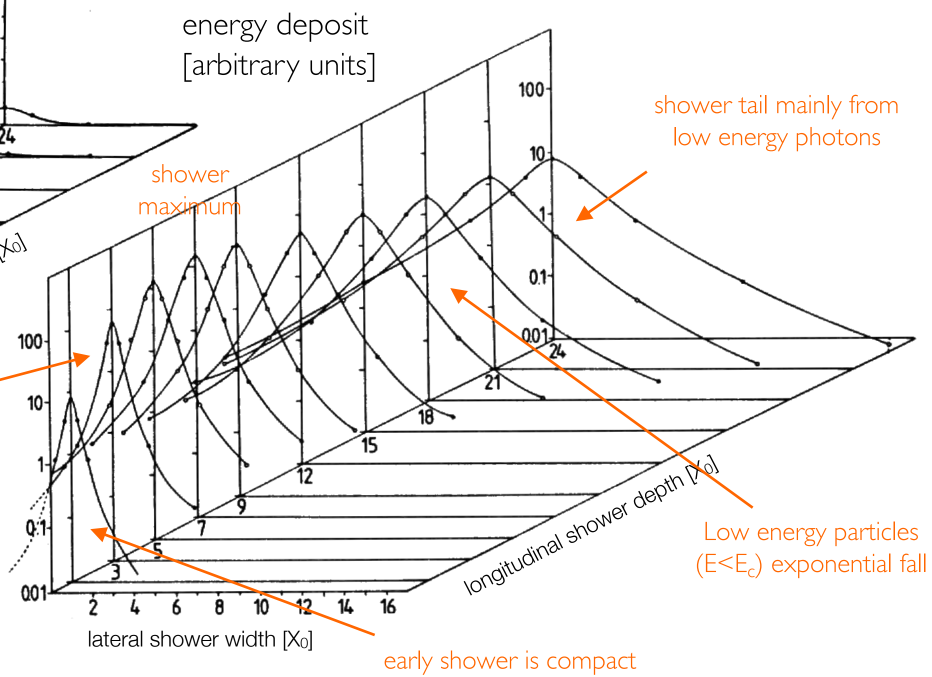
- Inner part: Coulomb scattering
  - ✓ Electrons and positrons move away from shower axis due to multiple scattering …
- Outer part: Low energy photons
  - ✓ Photons (and electrons) produced in isotropic processes (Compton scattering, photo-electric effect) move away from shower axis; predominant beyond shower maximum, particularly in high-Z absorber media…
- **Shower gets wider at larger depth**

EM shower profile summary

Longitudinal and transversal shower profile for a 6 GeV electron in lead absorber ...

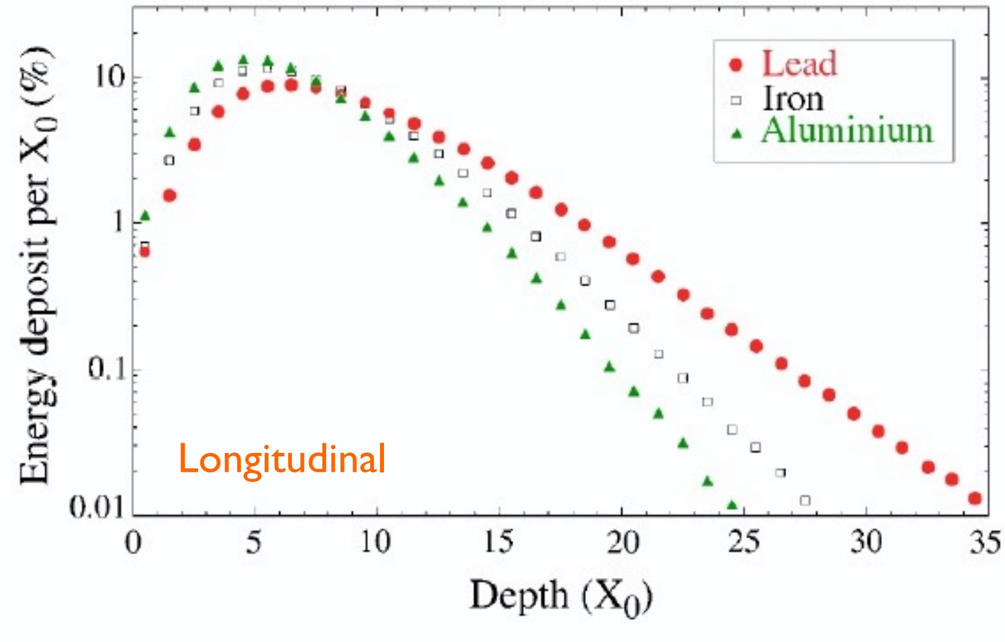
[left: linear scale; right: logarithmic scale]

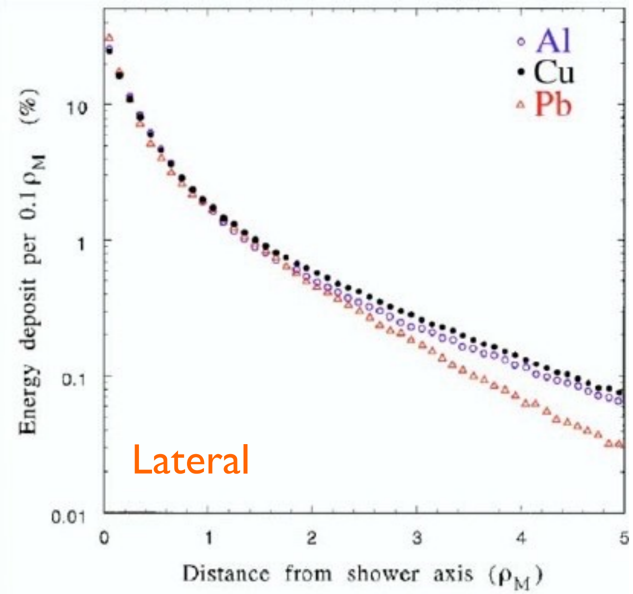




Low energy photons define real size of the shower: Compton and photon-electric effects spread the shower further

EM shower development dependence on material



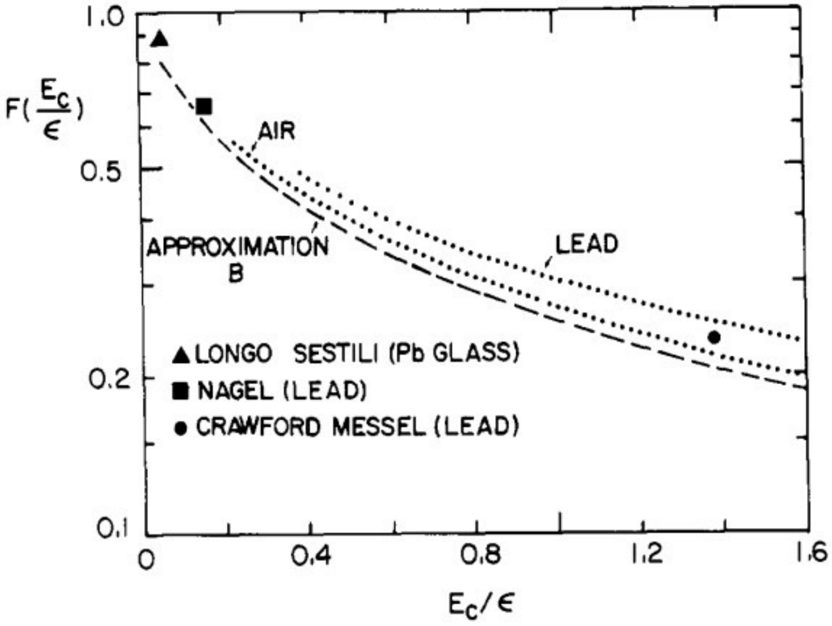


Scaling with X_0 is not perfect bot for longitudinal and lateral development: particle multiplication continues up to lower energies in high Z material (as E_c proportional to Z^{-1}), where Compton and photoelectron production become important: multiplication decreases more slowly

Response dependence on *detectable* energy cutoff

Total charged track length (g/cm2) $\propto X_0 E_0/E_c$

- When $E_e < E_c$ electrons and positrons in shower don't produce photons anymore, and begin (primarily) to ionize medium
- These energy deposits generate the detector signal, if they can get converted to something "readable" (e.g. light or electrical current)
- E_{cutoff} = minimum kinetic energy of an electron (positron) that can be *detected* in a calorimeter
  - ✓ If $E_{cutoff} = 0$ total track length is fully detected in the calorimeter
  - ✓ For increasing values of E_{cutoff} (= ε in plot) fraction F of total track length detected on average in fully contained electromagnetic shower decreases



2.4

From Particle Physics To Calorimeter Design

Why this physics matters: design considerations and "rules"

- **The three key parameters from particle physics that drive calorimeter design**
  - ✓ Radiation length X_0 → sets the physical size of the EM calorimeter
  - ✓ Critical energy E_c → determines the shower energy scale and signal yield
  - ✓ Moliere radius R_M → sets the transverse granularity requirement

$$X_0 \propto \frac{A}{Z^2}$$

$$E_c \propto \frac{1}{Z}$$

$$R_M \propto \frac{A}{Z}$$

- **Design rules of thumb**
  - ✓ EM calorimeter depth: 25 X_0 contains >99% of EM shower energy
    - CMS ECAL: 25.8 X_0 PbWO4 crystals
    - ATLAS LAr: 22-26 X_0 longitudinal segmentation
  - ✓ Lateral containment: 3.5 R_M contains >99% of shower energy
  - ✓ Calorimeter cell size: should be comparable to R_M for good position resolution

- **Issue related to poor shower containment**
  - ✓ Longitudinal leakage: energy deposited beyond the back face of the calorimeter
    - Leakage fluctuations directly worsen energy resolution (more on this in dedicated lecture)
  - ✓ Lateral leakage: energy deposited outside the *cluster* reconstruction window
    - Affects energy linearity and resolution (more on this in dedicated lecture)

Radiation length, critical energy, Moliere radius

https://pdg.lbl.gov

Material	X_0 [cm]	R_M [cm]	E_c [MeV]	Z_{eff}	ρ [g/cm³]
PbWO4 (CMS ECAL)	0.89	2.0	9.5	73	8.28
BGO (L3)	1.12	2.3	10.1	66	7.13
NaI(Tl) (Crystal Ball)	2.59	5.3	10.2	50	3.67
CsI(Tl) (BaBar)	1.85	3.7	10.4	54	4.51
LYSO	1.14	2.0	12.1	66	7.1
LAr (ATLAS)	14	9.5	31	18	1.4
LKr (NA48)	4.79	6.3	16	36	2.42
Pb	0.56	1.6	7.2	82	11.35
Fe	1.76	1.76	21	26	7.87
Cu	1.43	1.6	18.8	29	8.96
W	0.35	0.9	8	74	19.3
U	0.32	0.9	7.4	92	19.1
Scintillator (BC-408)	42.4	10.6	84	6	1.032
Si	9.4	4.9	40	14	2.33

What did we learn today?

- **Week 1 (Foundations)**
  - ✓ Lecture 1: Why calorimetry?
  - ✓ Lecture 2: EM shower physics
    - *2.1 Photon Interactions in Matter*
      - *Three dominant processes (photoelectric, Compton, pair production) at different E and Z; pair production introduces radiation length X_0 as key EM scale*
    - *2.2 Charged-Particle Interactions in Matter*
      - *"Heavy" particle: mostly ionization unless ultra-relativistic*
      - *Electrons: bremsstrahlung above E_c lead to EM shower; mostly ionization below E_c; energy-loss fluctuations (Landau distribution) matter for thin active layers*
    - *2.3 Electromagnetic Shower development*
      - *Concurrent pair production and bremsstrahlung, both governed by radiation length*
      - *Longitudinal depth scales logarithmically with energy; transverse size (Molière radius) energy-independent*
      - *The devil is in the details: some deviation from simple X_0 scaling to be accounted for*
    - *2.4 From Particle Physics To Calorimeter Design*
      - *X_0, E_c and R_M have different dependence on A and Z, to be accounted for in detector design*
      - *Shower containment rules (25 X0 depth, 3.5 RM lateral) directly constrain calorimeter geometry (TBC)*